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Fighting Against Currency Depreciation, Macroeconomic Instability and Sudden Stops*

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Abstract

In this paper we show that in the aftermath of a crisis, a government that changes the nominal interest rate in response to currency depreciation can induce aggregate instability in the economy by generating self-fulfilling endogenous cycles. In particular if a government raises the interest rate proportionally more than an increase in currency depreciation then it induces self-fulfilling cyclical equilibria that are able to replicate some of the empirical regularities of emerging market crises. We construct an equilibrium characterized by the self-validation of people's expectations about currency depreciation and by the following stylized facts of the "Sudden Stop" phenomenon: a decline in domestic production and aggregate demand, a significantly larger currency depreciation, a collapse in asset prices, a sharp correction in the price of traded goods relative to non-traded goods and an improvement in the current account deficit.

Keywords: Small Open Economy, Interest Rate Rules, Currency Depreciation, Multiple Equilibria, Sudden Stops, Collateral Constraints.

JEL Classifications: E32, E52, E58, F41.

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1 Introduction

One of the most controversial issues that emerged with the Asian Crisis of 1997 was the appropriate interest rate policy to fight against currency depreciation. There was a debate between two opposite views. On one hand, the IMF advocated for higher interest rates to prevent excessive currency depreciation. They claimed that this policy could reduce capital outflows by raising the cost of currency speculation, and induce capital inflows by making domestic assets more attractive. This would restore the confidence in domestic currencies and stop their accelerated depreciation.[1] On the other hand, some critics of the IMF policy prescription argued that this policy would exacerbate the depreciation process.[2] They argued that raising interest rates could aggravate the recession that the Asian economies were sliding into and weaken significantly the balance sheets of the banking and corporate sectors. This in turn would generate expectations of future financial crises, external debt defaults, and currency depreciations. Hence the critics recommended to lower interest rates.

Although these views prescribed opposite policy recommendations, both views had something in common: to some extent they conceived the interest rate policy as a reaction function. They advocated for changes in the nominal interest rate "in response to" some macroeconomic indicators such as currency depreciation. In fact some of the theoretical and empirical works motivated by the debate have modelled, implicitly or explicitly, the interest rate policy as a feedback rule responding to some measure of nominal depreciation.[3] After all this oversimplified reaction function captures both the concern of the Asian economies about currency undervaluation and the use of the interest rate as an exclusive instrument to fight against currency depreciation.

Little is known about the macroeconomic consequences of these interest rate feedback policies in countries that have been hit by a crisis. In this paper we study some of the possible consequences. Our main result is that in the aftermath of a crisis, an interest rate policy that calls for changing the nominal interest rate in response to currency depreciation can induce aggregate instability in the economy by generating self-fulfilling endogenous cycles. In other words, this policy can cause cycles in the economy that are driven by people's self-fulfilling expectations and not by fundamentals.

Surprisingly this result holds both when a government raises or when it lowers the interest rate in response to an increase in the nominal depreciation rate. In both cases this policy induces a continuum of self-fulfilling cyclical equilibria. But if a government raises the interest rate proportionally more than the increase in currency depreciation then it induces self-fulfilling cyclical equilibria that are able to replicate some of the empirical regularities of emerging market crises. In particular we construct an equilibrium characterized by the self-validation of people's expectations about currency depreciation and by the following stylized facts

[1] See Stanley Fischer (1998) among others.

[2] See for instance Furman and Stiglitz (1998) and Radelet and Sachs (1998) among others.

[3] See for instance Cho and West (2001), Goldfajn and Baig (1998), and Lahiri and Vegh (2003), among others.

labelled by Calvo (1998) as the “Sudden Stop” phenomenon: a decline in domestic production and aggregate demand, a significantly larger currency depreciation, a collapse in asset prices, a sharp correction in the price of traded goods relative to non-traded goods and an improvement in the current account deficit.[4]

We derive these results in the context of a typical small open economy model with incomplete financial markets and traded and non-traded goods. We augment this model by adding some features that have been proved to be useful in explaining some of the stylized facts of the aftermath of a crisis. First, in accord with Burnstein, Eichenbaum and Rebelo (2005a,b) we introduce slow adjustment in the price of the non-traded good and non-traded distribution services for the traded good. These characteristics are crucial in explaining the large movements in real exchange rates after large devaluations. Second, following Christiano, Gust and Roldos (2004) we assume that firms require working capital to hire labor and international working capital to purchase an imported intermediate input. These features become important to obtain a decline in output when interest rates rise in the midst of a crisis.[5] And third as in the new literature about currency crises we introduce a collateral constraint: international loans must be guaranteed by physical assets such as capital.[6] This provides us with the following definition of a crisis. A crisis is a time when the constraint is binding.

It is possible to provide a basic explanation of why the previously mentioned policy rule can induce self-fulfilling fluctuations, although non-cyclical, in a sticky-price economy that in “good times” does not face a collateral constraint. In theory, the Uncovered Interest Parity (UIP) condition together with the policy rule, that links the nominal interest rate to the current depreciation rate, determine the dynamics of the nominal depreciation rate and the nominal interest rate.[7] Both are determined independently of the dynamics of other nominal and real variables of the economy. As a consequence of this we can construct the following self-fulfilling equilibrium. Suppose that in response to a sunspot, agents in the economy expect a higher non-traded goods inflation rate. Since the monetary authority does not react to these expectations then the real interest rate measured in terms of the non-traded inflation can decline. In response, households increase desired consumption of non-traded goods which leads firms to rise their prices. But by doing this, firms end validating the original non-traded inflation expectations.

Although appealing this intuition is incomplete unless we show that all the equilibrium conditions of the economy in “good times” are satisfied on the entire equilibrium path. We accomplish this goal in Section 2 of the paper. We consider a simple model that abstracts from the collateral constraint, distribution services and loan requirements but considers price stickiness for non-traded goods. The importance of considering this simple set-up is that, under some extra assumptions about the household’s utility function, it allows us

[4] Rigorously Calvo (1998) refers to a “reversal” of the current account deficit.

[5] See also Lahiri and Vegh (2002).

[6] This idea of modelling the crisis as an unexpected binding collateral constraint captures the essence of the “Sudden Stops.” For works that introduce collateral constraints see Braggion, Christiano and Roldos (2005), Caballero and Krishnamurthy (2001), Christiano, Gust and Roldos (2004), Krugman (1999), Mendoza and Smith (2002), and Paasche (2001) among others.

[7] As we will see in Section 3, for our results it is not necessary to assume that the UIP condition holds. Here we assume it to highlight the basic intuition.

to derive *analytical* results. We can show that interest rate policies that respond to currency depreciation rates can induce real indeterminacy or multiple equilibria in the economy.[8]

These analytical results are useful for two reasons. First and in contrast to the explanation provided above, we can use them to construct self-fulfilling equilibria that are based on expectations of a different variable from the non-traded inflation such as currency depreciation. Second they are useful in making clear the main features of the model that explain the presence of real indeterminacy. They are the following: the interest rate policy, the presence of price-stickiness for non-traded goods and the exclusive dependence of the policy on currency depreciation. In Section 2 we elaborate on their interaction and their roles in our results.

Once we introduce a binding collateral constraint, distribution services and the need of working capital to hire productive factors, it is not longer possible to characterize analytically the equilibrium of the economy. We have to rely on numerical simulations for a sensible calibrated version of this augmented model. Nevertheless the self-fulfilling equilibrium mechanism explained before is still at the heart of the results from these simulations. As we show in Section 3 of the paper the simulations confirm the results of the simple model: interest rate policies that respond to currency depreciation rates are prone to induce multiple equilibria. But in this case, as mentioned before, these equilibria are cyclical because of the binding collateral constraint. This is not surprising since the seminal work by Kiyotaki and Moore (1997) shows that a binding collateral constraint induces credit cycles that may amplify business cycles in an economy. In our work this constraint in tandem with the previously mentioned policy rule and the presence of price stickiness is what leads to "endogenous cycles" or multiple cyclical equilibria.

A reader that is familiarized with the interest rate rules literature in closed and open economies may find an interesting connection between our results and this literature.[9] This connection poses the question of whether a rule that reacts aggressively and positively to past CPI-inflation and aggressively or timidly to current depreciation will preclude the previous multiple equilibria results. After all the interest rate literature claims that aggressive rules to past inflation are more likely to guarantee a unique equilibrium. In Section 3 we show that even in the augmented model such a policy can still induce real indeterminacy as long as the rule responds positively or negatively to current depreciation. Therefore it is the response to depreciation what opens the possibility of real indeterminacy.

We believe our results are novel and important because of their implications. First they provide a possible explanation of why the empirical literature has not been able to obtain conclusive evidence about whether higher interest rates can cause nominal exchange depreciation or instead appreciation in the aftermath of

[8] From now on we will use the terms "multiple equilibria" and "real indeterminacy" (a "unique equilibrium" and "real determinacy") interchangeably. By real indeterminacy we mean a situation in which the behavior of one or more (real) variables of the economy is not pinned down by the model. This situation implies that there are multiple equilibria, which in turn opens the possibility of having fluctuations in the economy generated by endogenous beliefs that are of the sunspot type; i.e., they are based on stochastic variables that are extrinsic in Cass and Shell's (1983) terminology.

[9] See for instance Benhabib et al. (2001), Taylor (1999), and Woodford (2003) among others. See also Zanna (2003) for a determinacy of equilibrium analysis for interest rate rules in small open economies.

a crisis.[10] This literature has tried to control for the variables that influence the nominal exchange rate. But our results suggest that there can be potential influences that may depend on "sunspots" which in turn can induce self-fulfilling cycles in the nominal exchange rate (or the nominal depreciation rate) as well as in other variables. Clearly these influences do not depend on fundamentals and their effect is something that the empirical literature should take into account.

Second, to the extent that these interest rate policies can induce multiple self-fulfilling equilibria in the economy then they can be costly in terms of macroeconomic instability and welfare. In other words these policies can lead to "sunspot" equilibria that are characterized by a large degree of volatility of some macroeconomic aggregates such as consumption; and provided that agents are risk averse, then these rules can induce equilibria where welfare can decrease. This has not been studied in the previous literature on interest rate policies during and in the aftermath of the Asian crisis. For instance, Lahiri and Vegh (2000, 2003) and Flood and Jeanne (2000) focus on the fiscal and output costs of higher interest rates before and after a crisis. In addition Lahiri and Vegh (2003) claim that there is a non-monotonic relationship between welfare and the increase in interest rates. Whereas Christiano, Gust and Roldos (2004) and Braggion, Christiano and Roldos (2005) explore conditions under which a cut (rise) in the interest rate in the midst of a crisis will stimulate output and employment and improve welfare.

Third our results cannot be understood as arguments that favor a particular view of the previously mentioned debate. They represent a simple example of policy induced macroeconomic instability regardless of whether the government increases or decreases the interest rate in response to currency depreciation. What is crucial in our analysis is the feedback response of the nominal interest rate to nominal depreciation. In this regard, we unveil a peril that may be present in previously mentioned policy recommendations that has not been discussed before.

Fourth by constructing a self-fulfilling equilibrium that replicates some of the stylized facts of the "Sudden Stops" we suggest that these interest rate policies may have contributed to generating the dynamic cycles experienced by the Asian economies in the aftermath of the crisis.

The remainder of this paper is organized as follows. In Section 2 we consider the simple set-up of the economy in "good times" and pursue analytically the characterization of the equilibrium for the interest rate policy that responds to currency depreciation. In section 3 we add to the simple model a collateral constraint, non-traded distribution services and the loan requirements to hire factors of production. Since we assume that the collateral constraint is binding, then this augmented set-up represents the economy in the aftermath of a crisis or the economy in "bad times." Through a calibrated simulation of the economy we pursue the determinacy of equilibrium analysis and confirm the results derived in Section 2. In section 4 we use this augmented set-up and construct a self-fulfilling equilibrium that captures the stylized facts of a

[10] See the review of this literature by Montiel (2003). Some of the papers in this literature are Basurto and Ghosh (2000), Caporale et al. (2005), Cho and West (2003), Dekle et al. (2001,2002), Furman and Stiglitz (1998), Goldfajn and Baig (1998), Goldfajn and Gupta (1999), and Gould and Kamin (2000) among others.

"Sudden Stop." Finally in Section 5 we present some concluding remarks.

2 The Simple Model: The Economy in "Good Times"

In this section we develop a simple infinite-horizon small open economy model. The economy is populated by a continuum of identical household-firm units and a government who are blessed with perfect foresight. Before we describe in detail the behavior of these agents we state a few general assumptions and definitions.

There are two consumption goods: a traded good and a composite non-traded good whose prices are denoted by P_t^T and P_t^N respectively. For this simple model we assume that the law of one price holds for the traded good. Then $P_t^T = \mathcal{E}_t P_t^{T*}$ where $\mathcal{E}_t$ is the nominal exchange rate and P_t^{T*} is the foreign price of the traded good. Later we will relax this assumption. We also normalize the foreign price of the traded good to one implying that $P_t^T = \mathcal{E}_t$.

The real exchange rate (e_t) is defined as the ratio between the price of traded goods and the aggregate price of non-traded goods, $e_t = \mathcal{E}_t / P_t^N$. From this definition we deduce that

$$e_t = e_{t-1}\left(\frac{\epsilon_t}{\pi_t^N}\right) \tag{1}$$

where $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1}$ is the gross nominal depreciation and $\pi_t^N = P_t^N / P_{t-1}^N$ is the gross non-traded inflation.

2.1 The Government

The government issues two nominal liabilities: money, M_t^g, and a domestic bond, B_t^g, that pays a gross nominal interest rate R_t. It does not have access to foreign debt and makes lump-sum transfers to the household-firm units, τ_t, pays interest on its domestic debt, $(R_t - 1)B_t^g$, and receives revenues from seigniorage. Its budget constraint is described by $m_t^g + b_t^g = \frac{m_{t-1}^g}{\epsilon_t} + \tau_t + \frac{R_{t-1} b_{t-1}^g}{\epsilon_t}$ where $m_t^g = \frac{M_t^g}{\mathcal{E}_t}$ and $b_t^g = \frac{B_t^g}{\mathcal{E}_t}$.

We assume that the government follows a Ricardian fiscal policy. That is, the government picks the path of the lump-sum transfers, τ_t, in order to satisfy the intertemporal version of its budget constraint in conjunction with the transversality condition $\lim\limits_{t\to\infty} \frac{b_t^g}{\prod_{s=0}^{t-1}\left(\frac{R_s}{\epsilon_{s+1}}\right)} = 0$.

On the other hand monetary policy is described as an interest-rate feedback rule whereby the government maneuvers the nominal interest rate of the domestic bond in response to currency depreciation. As we mentioned earlier, the motivation of this rule comes from the debate about the appropriate interest rate policy to fight against currency depreciation in the aftermath of the Asian crisis. To some extent the diverse policy recommendations conceived the interest rate policy as a reaction function. In fact some of the works inspired by this debate such as Cho and West (2001), Goldfajn and Baig (1998), and Lahiri and Vegh (2003) among others, have already considered describing the interest rate policy, implicitly or explicitly, as a rule that reacts to some measure of nominal depreciation.

Specifically we assume that the government can implement the following rule

$$R_t = \bar{R}\rho\left(\frac{\epsilon_t}{\bar{\epsilon}}\right) \tag{2}$$

where $\rho(.)$ is a continuous, differentiable and strictly positive function in its argument with $\rho(1) = 1$ and $\rho_\epsilon \equiv \rho'(1) \neq 0$; and $\bar{R}$ and $\bar{\epsilon}$ are the targets of the nominal interest rate and the nominal depreciation rate that the government wants to achieve.[11] In this sense the rule responds to the deviation of the current depreciation rate from the depreciation target.[12]

We also assume that the rule can respond positively to the deviation of the nominal depreciation rate from its target, $\rho_\epsilon > 0$, capturing the policy recommendation of the IMF policy makers; or it can react negatively, $\rho_\epsilon < 0$, describing, to some extent, the policy recommendations of the opposite view. Nevertheless we exclude the cases $\rho_\epsilon = -1, 1$.[13] In other words the interest rate policy corresponds to (2) with $\rho_\epsilon \equiv \rho'(1) \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$.

2.2 The Household-Firm Unit

There is a large number of identical household-firm units. They have perfect foresight, live infinitely and derive utility from consuming, not working and liquidity services of money. The intertemporal utility function of the representative unit is described by

$$\sum_{t=0}^{\infty}\beta^t\left[U(c_t^T) + V(c_t^N) + H(h_t^T) + L(h_t^N) + J(m_t)\right] \tag{3}$$

where $\beta \in (0, 1)$ corresponds to the discount rate, c_t^T and c_t^N denote the consumption of traded and non-traded goods respectively, h_t^T and h_t^N are the labor allocated to the production of the traded good and the non-traded good, and m_t refers to real money holdings measured with respect to foreign currency. The specification in (3) assumes separability in the single period utility function among consumption, labor and real money balances. By doing this we remove the distortionary effects of transactions money demand.[14] Moreover we introduce separability in the utility derived from c_t^T, c_t^N, h_t^T and h_t^N which will allow us to

[11] For simplicity we also assume that these targets correspond to the steady-state levels of these variables.

[12] Below we will consider other interest rate policies that differ in terms of the timing of the rule and on the inclusion of other arguments such as the CPI-inflation rate.

[13] The reason of this is that our analysis relies on a loglinearized system of equations that describes the dynamics of the economy. The cases of $\rho_\epsilon = 1$ or $\rho_\epsilon = -1$ introduce a unit root in this system precluding the possibility of using this system to pursue a meaningful determinacy of equilibrium analysis.

[14] Because of this we can write the real money balances that enter the utility function in terms of foreign currency, $m_t \equiv \frac{M_t}{\mathcal{E}_t}$, without consequences for our results.

derive *analytical* results in the determinacy of equilibrium analysis. To complete the characterization of the utility function we also make the following assumption.

Assumption 1. *a) $U(.)$, $V(.)$, $H(.)$, $L(.)$ and $J(.)$ are continuous and twice differentiable; and b) $U(.)$, $V(.)$, and $J(.)$ are strictly increasing ($U_T \equiv \frac{dU}{dc_t^T} > 0$, $V_N > 0$, $J_m > 0$) and strictly concave ($U_{TT} < 0$, $V_{NN} < 0$, $J_{mm} < 0$) whereas $H(.)$ and $L(.)$ are strictly decreasing ($H_T \equiv \frac{dH}{dh_t^T} < 0$, $L_N < 0$) and concave ($H_{TT} \leq 0$, $L_{NN} \leq 0$).*

The representative household-firm unit is engaged in the production of a flexible-price traded good and a sticky-price non-traded good by employing labor from a perfectly competitive market. The technologies are described by

$$y_t^T = F\left(\breve{h}_t^T\right) \qquad \textit{and} \qquad y_t^N = G\left(\tilde{h}_t^N\right)$$

where $\breve{h}_t^T$ and $\tilde{h}_t^N$ denote the labor hired by the household-firm unit for the production of the traded good and the non-traded good respectively. The technologies satisfy the following assumption.

Assumption 2. *$F(.)$ and $G(.)$ are continuous, twice differentiable, strictly increasing ($F_T \equiv \frac{dF}{dh_t^T} > 0$, $G_N > 0$), and strictly concave ($F_{TT} < 0$, $G_{NN} < 0$).*

Consumption of the non-traded good, c_t^N, is a composite good made of a continuum of intermediate differentiated goods. The aggregator function is of the Dixit-Stiglitz type. Each household-firm unit is the monopolistic producer of one variety of non-traded intermediate goods. The demand for the intermediate good is of the form $C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ satisfying $d(1) = 1$ and $d'(1) = -\mu$ with $\mu > 1$ where C_t^N denotes the level of aggregate demand for the non-traded good, $\tilde{P}_t^N$ is the nominal price of the intermediate non-traded good produced by the household-firm unit and P_t^N is the price of the composite non-traded good. The unit that behaves as a monopolist in the production of the non-traded good sets the price of the good it supplies, $\tilde{P}_t^N$, taking the level of aggregate demand for the good as given. Specifically the monopolist is constrained to satisfy demand at that price. That is

$$G\left(\tilde{h}_t^N\right) \geq C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) \tag{4}$$

Following Rotemberg (1982) we introduce nominal price rigidities for the intermediate non-traded good. The household-firm unit faces a resource cost of the type $\frac{\gamma}{2}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}^N\right)^2$, that reflects that it is costly having the price of the good that it sets grow at a different rate from $\bar{\pi}^N$, the steady-state level of the gross non-traded inflation rate.

There are incomplete markets. The representative household-firm unit has access to two different risk free bonds: a domestic bond issued by the government, B_t, that pays a gross nominal interest rate, R_t and a foreign bond, b_t^*, that pays a gross foreign interest rate R_t^*. In addition, the unit receives a wage income from working, $W_t\left(h_t^T + h_t^N\right)$, lump-sum transfers from the government, τ_t, and dividends from selling the traded good and the non-traded composite good. Then its flow budget constraint in units of the traded good can be written as

$$m_t + b_t \leq \frac{m_{t-1}}{\epsilon_t} + \frac{R_{t-1}b_{t-1}}{\epsilon_t} + w_t\left(h_t^T + h_t^N\right) + \tau_t + \Omega_t - c_t^T - \frac{c_t^N}{e_t} \tag{5}$$

where $b_t = \frac{B_t}{\mathcal{E}_t}$, $w_t = \frac{W_t}{\mathcal{E}_t}$ and[15]

$$\Omega_t = \left[F\left(\check{h}_t^T\right) - w_t \check{h}_t^T\right] - \frac{1}{e_t}\left[\frac{\tilde{P}_t^N}{P_t^N} C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) - e_t w_t \tilde{h}_t^N - \frac{\gamma}{2}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}\right)^2\right] - R_{t-1}^* b_{t-1}^* + b_t^* \tag{6}$$

Equation (5) says that the end-of-period real financial domestic assets (money plus domestic bond) can be worth no more than the real value of financial domestic wealth brought into the period plus the sum of wage income, transfers and dividends (Ω_t) net of consumption. The dividends described in (6) correspond to the difference between sale revenues and costs, taking into account that through the firm-side the representative unit can hold foreign debt, b_t^*. For holdings of foreign debt the agent pays interests, $(R_{t-1}^* - 1)b_{t-1}^*$.

Besides the budget constraint the household-firm unit is subject to an Non-Ponzi game condition

$$\lim_{t\to\infty} \frac{n_t}{\prod_{s=0}^{t-1} R_s^*} \geq 0 \tag{7}$$

where $n_t = b_t + m_t - b_t^*$.

The representative household-firm unit chooses the set of sequences $\{c_t^T,\ c_t^N,\ h_t^T,\ h_t^N,\ \check{h}_t^T,\ \tilde{h}_t^N,\ \tilde{P}_t^N,\ b_t^*,\ b_t,\ m_t\}_{t=0}^{\infty}$ in order to maximize (3) subject to (4), (5), (6) and (7), given the initial condition n_{-1} and the set of sequences $\{R_t^*,\ R_t,\ \epsilon_t,\ e_t,\ P_t^N,\ w_t,\ \tau_t,\ C_t^N\}$. Note that since the utility function specified in (3) implies that the preferences of the agent display non-sasiation then both constraints (5) and (7) hold with equality. The Appendix contains a detailed derivation of the necessary conditions for optimization. Imposing these conditions along with the market clearing conditions in the labor market, the equilibrium symmetry ($\tilde{P}_t^N = P_t^N$ and $\tilde{h}_t^N = h_t^N$), the market clearing condition for the non-traded good

$$G\left(h_t^N\right) = c_t^N + \frac{\gamma}{2}\left(\pi_t^N - \bar{\pi}^N\right)^2 \tag{8}$$

and the definitions $\pi_t^N = P_t^N / P_{t-1}^N$, $d(1) = 1$ and $d'(1) = -\mu$ we obtain

$$R_t^* = \frac{R_t}{\epsilon_{t+1}} \tag{9}$$

$$-\frac{H_T(h_t^T)}{U_T(c_t^T)} = F_T\left(h_t^T\right) \tag{10}$$

[15] By having only one real wage w_t for h_t^T and h_t^N we are implicitly assuming that there is perfect labor mobility between the production of traded and non-traded goods.

$$U_T(c_t^T) = \beta R_t^* U_T(c_{t+1}^T) \tag{11}$$

$$V_N(c_t^N) = \frac{\beta R_t}{\pi_{t+1}^N} V_N(c_{t+1}^N) \tag{12}$$

$$\frac{V_N(c_{t+1}^N)\left(\pi_{t+1}^N - \bar{\pi}^N\right)\pi_{t+1}^N}{V_N(c_t^N)} = \frac{\left(\pi_t^N - \bar{\pi}^N\right)\pi_t^N}{\beta} + \frac{\mu c_t^N}{\beta\gamma}\left(\frac{\mu - 1}{\mu} - mc_t\right) \tag{13}$$

where $mc_t = -\frac{L_N(h_t^N)}{V_N(c_t^N)G_N\left(h_t^N\right)}$ corresponds to the marginal cost of producing the non-traded good. In addition equilibrium in the traded good market implies that

$$b_t^* - b_{t-1}^* = \left(R_{t-1}^* - 1\right) b_{t-1}^* + c_t^T - F\left(h_t^T\right) \tag{14}$$

The interpretation of these equations is straightforward. Condition (9) corresponds to an Uncovered Interest Parity condition (UIP) that equalizes the returns of the foreign and domestic bonds. Equation (10) makes the marginal rate of substitution between labor (assigned to the production of the traded good) and consumption of the traded good equal to the marginal product of labor in the production of the traded good. Equations (11) and (12) are the standard Euler equations for consumption of the traded good and consumption of the non-traded good. Equation (13) corresponds to the augmented Phillips curve for the sticky-price non-traded goods inflation.[16] And (14) corresponds to the current account equation.

2.3 Capital Markets

We introduce imperfect capital markets using the following ad-hoc upward-sloping supply curve of funds on the world capital market

$$R_t^* = R^* f\left(\frac{b_t^*}{\bar{b}^*}\right) \quad \text{with} \quad f'\left(\frac{b_t^*}{\bar{b}^*}\right) > 0, \quad f(1) = 1, \quad f'(1) = \psi > 0, \tag{15}$$

where $f\left(\frac{b_t^*}{\bar{b}^*}\right)$ corresponds to the country-specific risk premium and R^* is the risk free international interest rate. This specification captures the idea that the small borrowing economy faces a world interest rate, R_t^*, that increases when the stock of the debt issued by the country, b_t^*, is above its long run level, $\bar{b}^*$. Then as the external debt grows so does the risk of default, and in order to compensate the lenders for this risk, the economy has to pay them a premium over the risk free international interest rate.

The reason for introducing (15) is merely technical. By doing so, we "close the small open economy" and avoid the unit root problem as discussed in Schmitt-Grohé and Uribe (2003). This will allow us to obtain meaningful results from the determinacy of equilibrium analysis once we log-linearize the equations of the

[16] We would have derived a similar augmented Phillips curve if we had follow Calvo's (1983) approach.

model.[17] Our results are invariant to other approaches to "close the small open economy" such as complete markets or convex portfolio adjustment costs.

Finally throughout this paper we will also assume that the long-run level of foreign stock of debt is positive as stated in the following assumption.

Assumption 3. *The long-run level of the foreign stock of debt is positive:* $\bar{b}^* > 0$.

2.4 A Perfect Foresight Equilibrium

We are ready to provide a definition of a perfect foresight equilibrium in this economy.

Definition 1 *Given the initial condition* b^*_{-1}, *the steady-state level of foreign debt* $\bar{b}^*$ *and the depreciation target* $\bar{\epsilon}$, *a symmetric perfect foresight equilibrium is defined as a set of sequences* $\{c_t^T,\ c_t^N,\ h_t^T,\ h_t^N,\ b_t^*,\ \epsilon_t,\ \pi_t^N,\ R_t,\ R_t^*\}_{t=0}^{\infty}$ *satisfying: a) the market clearing conditions for the non-traded and traded goods, (8) and (14), b) the UIP condition (9), c) the intratemporal efficient condition (10), d) the Euler equations for consumption of traded and non-traded goods, (11) and (12), e) the augmented Phillips curve, (13), f) the monetary policy (2) and g) the ad-hoc upward-sloping supply curve of foreign funds (15).*

Note that this definition ignores the budget constraint of the government and its transversality condition. The reason is that by following a Ricardian fiscal policy the government guarantees that the intertemporal version of its budget constraint in conjunction with its transversality condition will be always satisfied. In addition real money balances do not appear in the definition. This is because monetary policy is described as an interest rate rule and real balances enter in the utility function in a separable way. In fact once we solve for $\{c_t^T,\ c_t^N,\ h_t^T,\ h_t^N,\ b_t^*,\ \epsilon_t,\ \pi_t^N,\ R_t,\ R_t^*\}_{t=0}^{\infty}$ it is possible to retrieve the set of sequences $\{\lambda_t,\ e_t,\ m_t,\ b_t,\ w_t,\ mc_t\}_{t=0}^{\infty}$ using (1), (5), and equations (40), (41), (43), (45) and (47) that are presented in the Appendix.

2.5 The Determinacy of Equilibrium Analysis

In order to pursue the determinacy of equilibrium analysis we will log-linearize the system of equations that describe the dynamics of this economy around a steady state $\{\bar{c}^T,\ \bar{c}^N,\ \bar{h}^T,\ \bar{h}^N,\ \bar{b}^*,\ \bar{\epsilon},\ \bar{\pi}^N,\ \bar{R},\ \bar{R}^*\}$. In the Appendix we characterize this steady state.

Log-linearizing the equations of Definition 1 around the steady-state yields

$$\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t \qquad \text{with} \ \ \rho_\epsilon \neq 0 \text{ and either } \ |\rho_\epsilon| > 1 \text{ or } \ |\rho_\epsilon| < 1 \tag{16}$$

[17] The "unit-root problem" that is commonly present in small open economy models arises because of assuming that $R_t^* = \frac{1}{\beta}$. To see why, use this assumption together with condition (48) to deduce that $\lambda_t = \lambda_{t+1}$. This is an equation that has a unit root and that introduces a unit root in the entire dynamical system of the simple set-up. Then it is not valid to apply the common technique of linearizing the system around the steady state and studying the eigenvalues of the Jacobian matrix in order to characterize local determinacy of the dynamical system. See Schmitt-Grohé and Uribe (2003).

$$\hat{R}_t = \hat{R}^*_t + \hat{\epsilon}_{t+1} \tag{17}$$

$$\hat{c}^N_t = \hat{c}^N_{t+1} - \xi^N \left(\hat{R}_t - \hat{\pi}^N_{t+1} \right) \tag{18}$$

$$\hat{\pi}^N_t = \beta \hat{\pi}^N_{t+1} + \beta \varphi \hat{c}^N_t \tag{19}$$

$$\hat{b}^*_t = \left(\frac{1+\psi}{\beta} \right) \hat{b}^*_{t-1} + \kappa \hat{c}^T_t \tag{20}$$

$$\hat{c}^T_t = \hat{c}^T_{t+1} - \xi^T \left(\hat{R}_t - \xi^T \hat{\epsilon}_{t+1} \right) \tag{21}$$

where $\hat{x}_t = \log\left(\frac{x_t}{\bar{x}}\right)$ and

$$\xi^T = -\frac{U_T}{U_{TT}\bar{c}^T} > 0 \qquad \xi^N = -\frac{V_N}{V_{NN}\bar{c}^N} > 0 \qquad \sigma^T = \frac{H_T}{H_{TT}\bar{h}^T} > 0 \qquad \sigma^N = \frac{L_N}{L_{NN}\bar{h}^N} > 0$$

$$\omega^T = -\frac{F_T}{F_{TT}\bar{h}^T} > 0 \qquad \omega^N = -\frac{G_N}{G_{NN}\bar{h}^N} > 0 \tag{22}$$

$$\kappa = \frac{1}{\bar{b}^*}\left[\bar{c}^T + \frac{F_T \bar{h}^T \sigma^T \omega^T}{(\sigma^T + \omega^T)\,\xi^T}\right] > 0 \text{ and } \varphi = \left[\frac{(\mu-1)\bar{c}^N}{\beta\gamma\bar{\epsilon}^2}\right]\left[\frac{\bar{c}^N(\sigma^N + \omega^N)}{G_N \bar{h}^N \sigma^N \omega^N} + \frac{1}{\xi^N}\right] > 0$$

whose signs are derived using Assumptions 1, 2 and 3. Equations (16)-(21) correspond to the reduced log-linear representations of the policy rule, the UIP condition, the Euler equation for consumption of the non-traded good, the augmented Phillips curve, the current account equation, and the Euler equation for consumption of the traded good, respectively.

Our main goal in this Section is to show that the rule in (16) is prone to induce multiple equilibria in the economy described by equations (17)-(21). Proving this implies that this policy can cause fluctuations in the economy that are driven by people's self-fulfilling beliefs and not by fundamentals. In fact, before we provide a formal proof of the existence of multiple equilibria, it is worth developing a simple intuition of why this policy rule can induce self-fulfilling equilibria. To do so it is sufficient to concentrate on equations (16)-(19) in order to construct the following argument.

Note that given the international interest rate, $\hat{R}^*_t$, then the policy rule (16) and the UIP condition (17) determine the dynamics of the depreciation rate, $\hat{\epsilon}_t$, and the nominal interest rate, $\hat{R}_t$. More importantly the nominal interest rate, $\hat{R}_t$, is not affected by either the non-traded good inflation, $\hat{\pi}^N_t$, or the consumption of the non-traded good, c^N_t. Taking this into account we can construct the following self-fulfilling equilibrium. Assume that agents in response to a sunspot expect a higher non-traded good inflation in the next period. Since the interest rate policy does not react to these expectations then the real interest rate measured with respect to the expected non-traded good inflation, $\hat{R}_t - \hat{\pi}^N_{t+1}$, declines. This stimulates consumption of the non-traded good according to (18). And as a response to this, firms raise the price of the non-traded good inducing a higher non-traded inflation as can be seen in (19). Hence the original beliefs of a higher non-traded good inflation are validated.

This simple intuition is appealing but incomplete unless we show that all the equilibrium conditions of the economy (16)-(21) are satisfied on the entire equilibrium path. In other words we need to characterize formally the equilibrium of this economy. To accomplish this goal we first manipulate equations (16)-(21) and write them in the matrix form

$$\begin{pmatrix} \hat{\epsilon}_{t+1} \\ \hat{b}^*_t \\ \hat{c}^T_{t+1} \\ \hat{\pi}^N_{t+1} \\ \hat{c}^N_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \rho_\epsilon & -\frac{\psi(1+\psi)}{\beta} & -\psi\kappa & 0 & 0 \\ 0 & \frac{1+\psi}{\beta} & \kappa & 0 & 0 \\ 0 & \frac{\psi(1+\psi)\xi^T}{\beta} & \left(1+\psi\kappa\xi^T\right) & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\beta} & -\varphi \\ \rho_\epsilon\xi^N & 0 & 0 & -\frac{\xi^N}{\beta} & \left(1+\varphi\xi^N\right) \end{pmatrix}}_{J^c} \begin{pmatrix} \hat{\epsilon}_t \\ \hat{b}^*_{t-1} \\ \hat{c}^T_t \\ \hat{\pi}^N_t \\ \hat{c}^N_t \end{pmatrix} \tag{23}$$

Then we use this system to find and to compare the dimension of the unstable subspace of the system to the number of non-predetermined variables.[18] If the dimension of this subspace is smaller than the number of non-predetermined variables then, from the results by Blanchard and Kahn (1980), we can infer that there exist multiple perfect foresight equilibria. This forms the basis for the existence of self-fulfilling fluctuations.

The following Proposition states the main result of the determinacy of equilibrium analysis: an interest rate policy that raises or lowers the nominal interest rate in response to current currency depreciation can lead to real indeterminacy, or, equivalently, to multiple equilibria.

Proposition 1 *If the government follows an interest rate policy rule such as $\hat{R}_t = \rho_\epsilon\hat{\epsilon}_t$ with $\rho_\epsilon \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$, then there exists a continuum of perfect foresight equilibria in which the sequences $\{\hat{\epsilon}_t,\ \hat{b}^*_t,\ \hat{c}^T_t,\ \hat{\pi}^N_t,\ \hat{c}^N_t\}_{t=0}^{\infty}$ converge asymptotically to the steady state. In addition*

> ***a)*** *if $|\rho_\epsilon| > 1$ then the degree of indeterminacy is of order 1.*[19]
>
> ***b)*** *if $|\rho_\epsilon| < 1$ then the degree of indeterminacy is of order 2.*

Proof. The eigenvalues of the matrix J^c in (23) correspond to the roots of the characteristic polynomial $\mathcal{P}^c(v) = |J^c - vI| = 0$. Using the definition of J^c in (23) this polynomial can be written as

$$\mathcal{P}^c(v) = (v - \rho_\epsilon)\,\mathcal{P}^f(v) = 0 \tag{24}$$

[18] The dimension of the unstable subspace is given by the number of roots of the system that are outside the unit circle. See Blanchard and Kahn (1980).

[19] The degree of indeterminacy is defined as the difference between the number of non-predetermined variables and the dimension of the unstable subspace of the log-linearized system.

where

$$\mathcal{P}^f(v) = \left[v^2 - \left(1 + \frac{1+\psi}{\beta} + \psi\kappa\xi^T\right)v + \frac{1+\psi}{\beta}\right]\left[v^2 - \left(1 + \frac{1}{\beta} + \varphi\xi^N\right)v + \frac{1}{\beta}\right]$$

By Lemma 4 in the Appendix we know that the characteristic polynomial $\mathcal{P}^f(v) = 0$ has real roots satisfying $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$ and $|v_4| > 1$. The fifth root of $\mathcal{P}^c(v) = 0$ is $v_5 = \rho_\epsilon$. Clearly if $|\rho_\epsilon| > 1$ then $|v_5| > 1$ whereas if $|\rho_\epsilon| < 1$ then $|v_5| < 1$. Therefore using this, the characterization of the roots of $\mathcal{P}^f(v) = 0$ and (24) we can conclude the following. If $|\rho_\epsilon| > 1$ then $\mathcal{P}^c(v) = 0$ has three explosive roots namely $|v_2| > 1$, $|v_4| > 1$ and $|v_5| > 1$. While if $|\rho_\epsilon| < 1$ then $\mathcal{P}^c(v) = 0$ has two explosive roots namely $|v_2| > 1$ and $|v_4| > 1$. Therefore regardless of whether $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$ there are at most three explosive roots. Given that there are four non-predetermined variables, $\hat{\epsilon}_t$, $\hat{c}_t^T$, $\hat{\pi}_t^N$ and $\hat{c}_t^N$, then the number of non-predetermined variables is greater than the number of explosive roots. Applying the results of Blanchard and Kahn (1980) it follows that there exists an infinite number of perfect foresight equilibria converging to the steady state.

Finally parts a) and b) follow from the difference between the number of non-predetermined variables and the number of explosive roots when $|\rho_\epsilon| > 1$ and $|\rho_\epsilon| < 1$ respectively. ■

Table 1: Determinacy of Equilibrium Analysis

	The Simple Model		*The Augmented Model*	
	Degree of Responsiveness		*Degree of Responsiveness*	
Interest Rate Policy	$\lvert\rho_\epsilon\rvert < 1$	$\lvert\rho_\epsilon\rvert > 1$	$\lvert\rho_\epsilon\rvert < 1$	$\lvert\rho_\epsilon\rvert > 1$
Forward-Looking				
$\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ *with* $\rho_\epsilon \neq 0$	*M*	*M*	*M*	*M*
Contemporaneous				
$\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ *with* $\rho_\epsilon \neq 0$	*M*	*M*	*M*	*M*
Backward-Looking				
$\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ *with* $\rho_\epsilon \neq 0$	*M*	*U*	*M*	*M or U*

Note: *M refers to multiple equilibria and U refers to a unique equilibrium*

Proposition 1 has two important implications. First provided that the fiscal policy is Ricardian, then the interest rate policy considered in this Proposition will not pin-down the level of the nominal exchange rate.[20] Hence the same policy also induces nominal indeterminacy of the exchange rate level. Second, there is

[20] A Non-Ricardian fiscal policy combined with the monetary policy under study will determine the level of the nominal

nothing in the characterization of the equilibrium that prevents us from constructing self-fulfilling equilibria that are based on expectations of a different variable from the non-traded inflation. For instance we can construct a self-fulfilling equilibrium driven by people's beliefs about currency depreciation. We will pursue this exercise in Section 4.

The results of Proposition 1 also pose the question of whether policies that respond exclusively to either the future depreciation rate (a forward-looking policy, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$) or to the past depreciation rate (a backward-looking policy, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$) can still induce multiple equilibria. The answer to this question is affirmative and the characterization of the equilibrium under these policies is provided in the Appendix.

In particular we find that forward-looking policies, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ with $\rho_\epsilon \neq 0$, can lead to multiple equilibria when the interest rate response coefficient to future depreciation satisfies either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$. On the contrary backward-looking policies, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ with $\rho_\epsilon \neq 0$, that are very aggressive with respect to past depreciation and satisfy $|\rho_\epsilon| > 1$ will guarantee a unique equilibrium whereas timid policies that satisfy $|\rho_\epsilon| < 1$, can lead to real indeterminacy. These results as well as the results from Proposition 1 are presented in Table 1 in the columns labeled as "The Simple Model."[21]

It is important to understand the features of the model that allow for the existence of multiple equilibria. After all by unveiling them in the simple model of this section will also help us to understand the results in the richer set-up of the next section. The crucial features are the following: the description of monetary policy as an interest rate feedback rule, the introduction of price-stickiness in non-traded goods and the exclusive dependence of the rule on currency depreciation. By Sargent and Wallace (1975) we know that the first feature by itself leads to nominal indeterminacy of the exchange rate level (price level) in a flexible price model under a Ricardian fiscal policy. The second characteristic together with the rule elucidate why nominal indeterminacy turns into real indeterminacy. And finally the first two features in tandem with the exclusive response of the rule to currency depreciation is what explains why, at least for forward-looking and contemporaneous rules, multiple equilibria arise regardless of the degree of responsiveness of the rule.[22]

Although the results of this section are interesting, it is clear that the model suffers from at least two drawbacks. On one hand there is no specific feature in the model that captures the fact that the economy is in a crisis. On the other hand some of the dynamics of consumption and inflation (of non-traded goods) that are supported as a self-fulfilling equilibrium are completely at odds with the stylized facts of a "Sudden Stop."

exchange rate if $|\rho_\epsilon| > 1$ but not if $|\rho_\epsilon| < 1$.

[21] Our results will not be affected if we model monetary policy as $R_t = \rho(\epsilon_{t+s})$ with $s = -1, 0, 1$. The reason is that in (2) we have assumed that the target depreciation rate coincides with the long run steady state depreciation rate and the determinacy of equilibrium analysis is pursued using a log-linearized version of the system of equations that describe the dynamics of the economy around the steady state. In addition our general results still hold if we describe monetary policy as $\Delta R_t = R_t - R_{t-1} = \varphi(\frac{\epsilon_{t+s}}{\bar{\epsilon}})$ with $s = -1, 0, 1$ and $\varphi(1) = 0$. This resembles the implicit descriptions in some of the empirical works such as Gould and Kamin (2000), Caporale, Cipollini and Demetriades (2005) and Dekle, Hsiao and Wang (2001, 2002).

[22] In Zanna (2003) we show that in this simple set-up in order to guarantee a unique equilibrium a rule must respond aggressively to the non-traded inflation but timidly to depreciation. A rule that responds aggressively to currency depreciation still opens the possibility of multiple equilibria regardless of its response to non-traded inflation.

In particular the intuition that we provided to construct a self-fulfilling equilibrium implies that consumption of non-traded goods and inflation are positively correlated. On the contrary, the typical stylized facts of a crisis suggest that they are negatively correlated: there is a strong decline in consumption accompanied by an increase in inflation. In order to correct these drawbacks we will enrich the current model with some features that have been proved to be useful in explaining some stylized facts of a crisis. This defines the objective of our next section.

3 The Augmented Model: The Economy in "Bad Times"

In this Section we introduce some features that will enrich the simple model in several dimensions. First we introduce distribution costs (services) for the traded good. This together with price stickiness are crucial to explain the large movements in real exchange rates after large devaluations. Second, we assume that the household-firm units require working capital (loans) to hire labor and international working capital (loans) to purchase an imported intermediate input. This characteristic is important to obtain a decline in output and demand in the midst of the crisis when interest rates rise. Third we impose a collateral constraint: international loans must be guaranteed by physical assets such as capital. This provides a definition of a crisis. A crisis is a time when the constraint is binding and the shadow price of the constraint is greater than zero. Fourth we assume non-separability in the utility function between the two types of consumption. This will guarantee that our previous results are not driven by the specific assumption of separability. We proceed to explain how we introduce these features in the model and their influence in the previous equations.

3.1 The Additional Features

As in Burnstein et al. (2003) we assume that the traded good needs to be combined with some non-traded distribution services before it is consumed. In order to consume one unit of the traded good it is required η units of the basket of differentiated non-traded goods. Let $\breve{P}_t^T$, P_t^T and P_t^N be the price in the domestic currency that the household-firm unit receives from producing and selling the traded good, the price that it pays to consume this good and the general price level of the basket of differentiated non-traded goods, respectively. Hence the consumer price of the traded good is simply $P_t^T = \breve{P}_t^T + \eta P_t^N$. And since PPP holds at the production level of the traded good ($\breve{P}_t^T = \mathcal{E}_t \breve{P}_t^{T*}$) and the foreign price of the traded good is normalized to one ($\breve{P}_t^{T*} = 1$), we have that $P_t^T = \mathcal{E}_t + \eta P_t^N$.

The production of the non-traded good is still demand determined by

$$G\left(\tilde{h}_t^N, K^N\right) \geq C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) + \eta C_t^T d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) \tag{25}$$

where $d(1) = 1$, $d'(1) = -\mu$, C_t^N denotes the level of aggregate demand for the non-traded good, $\tilde{P}_t^N$ is the nominal price of the intermediate non-traded good produced by the household-firm unit and C_t^T corresponds to the level of aggregate consumption of the traded good. But now the demand requirements come from two sources.[23] They come from consumption of non-traded goods $C_t^N d\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ that provide utility and from non-traded distribution services $\eta C_t^T d\left(\frac{\tilde{P}_t^N}{P_t^N}\right)$ that are necessary to bring one unit of the traded good to the household-firm unit. Note that we assume that there is no difference between non-traded consumption goods and non-traded distribution services. As a consequence, in equilibrium the basket of non-traded goods required to distribute traded goods will have the same composition as the non-traded basket consumed by the household-firm unit.

The introduction of the loan requirements and the collateral constraint in the model follows Christiano et al. (2004). The household-firm unit requires domestic loans to hire labor ($\breve{h}_t^T$ and $\tilde{h}_t^N$) and international loans to buy an imported input (I_t) that will be used in the production of the traded good. These loans are obtained at the beginning of the period and repaid at the end of the period. In this sense they represent short-term debt and differ from long-term foreign debt b_{t-1}^*. We do not model, however, the financial institutions that provide these loans. Instead we assume that the domestic loans are provided by the government whereas the foreign loans are supplied by foreign creditors.[24] For these loans the unit pays interests $(R_t-1)W_t(\breve{h}_t^T+\tilde{h}_t^N)$ and $(R_t^*-1)\tilde{P}_t^T I_t$ that are accrued between periods, where R_t is the domestic nominal interest rate and R^* is the international interest rate. The latter is assumed to be constant and equal to $\frac{1}{\beta}$.

In contrast to the simple model we assume that the production technology of the traded good needs labor ($\breve{h}_t^T$), an imported input (I_t) and capital (K^T). In addition, the technology for the non-traded goods requires labor ($\tilde{h}_t^N$) and capital (K^N). That is

$$y_t^T = F\left(\breve{h}_t^T, I_t, K^T\right) \quad \text{and} \quad y_t^N = G\left(\tilde{h}_t^N, K^N\right)$$

Furthermore, as in Christiano et al. (2004) and Mendoza and Smith (2002), among others, capital is assumed to be time-invariant, does not depreciate and there is no technology to making it bigger.

Under these new features the dividends that the household-firm unit receives can be written as

$$\begin{aligned}\Omega_t &= F\left(\breve{h}_t^T, I_t, K^T\right) + \frac{1}{e_t}\left[\frac{\tilde{P}_t^N}{P_t^N}G\left(\tilde{h}_t^N, K^N\right) - \frac{\gamma}{2}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}\right)^2\right] \\ &\quad - w_t R_t \breve{h}_t^T - w_t R_t \tilde{h}_t^N - R^* I_t - R^* b_{t-1}^* + b_t^*\end{aligned} \tag{26}$$

where $w_t = \frac{W_t}{\mathcal{E}_t}$ and $e_t = \frac{\mathcal{E}_t}{P_t^N}$.

To model the crisis we follow closely Christiano et al. (2004) by imposing a collateral constraint on the household-firm unit

[23] See Corsetti et al. (2005).

[24] To formalize this point we could introduce financial institutions in the model that behave in a perfectly competitive way and supply the aforementioned loans. This would not change our main results.

$$R^* b^*_{t-1} + R^* I_t + w_t R_t \left(\check{h}^T_t + \tilde{h}^N_t\right) \leq \phi \left(q^N_t K^N + q^T_t K^T\right) \tag{27}$$

where q^N_t and q^T_t represent the real value (in units of foreign currency) of one unit of capital for the production of the non-traded and traded goods respectively, and ϕ is the fraction of these stocks that foreign creditors accept as collateral. The constraint (27) says that the total value of foreign and domestic debt that the representative household-firm unit has to pay to completely eliminate the debt of the firm by the end of period t cannot exceed the value of the collateral. The crisis makes this constraint unexpectedly binding in every period henceforth without the possibility of being removed.

Finally we will assume non separability in the utility function between the two types of consumption. Instead of having $U(c^T_t) + V(c^N_t)$ as the specification in (3) we define $\tilde{U}(c^T_t, c^N_t)$. But we will still assume separability among consumption, labor and real money balances.

With all these features the problem of the representative household-firm unit does not change significantly with respect to the one in the simple model. But the collateral constraint represents an extra condition that affects the decisions of the representative agent. We proceed to study how this constraint in tandem with the other features influence the optimal conditions of a symmetric equilibrium.

3.2 The New Equilibrium Conditions

The problem that the household firm unit has to solve is similar to the one presented in the simple model. The agent chooses the set of sequences $\{c^T_t, c^N_t, h^T_t, h^N_t, \check{h}^T_t, \tilde{h}^N_t, I_t, \tilde{P}^N_t, b^*_t, b_t, m_t\}_{t=0}^{\infty}$ in order to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\tilde{U}(c^T_t, c^N_t) + H(h^T_t) + L(h^N_t) + J(m_t)\right]$$

subject to the budget constraint

$$m_t + b_t \leq \frac{m_{t-1}}{\epsilon_t} + \frac{R_{t-1} b_{t-1}}{\epsilon_t} + w_t(h^T_t + h^N_t) + \tau_t + \Omega_t - \left(1 + \frac{\eta}{e_t}\right) c^T_t - \frac{c^N_t}{e_t}$$

and the constraints (7), (25), (26), and (27), given the initial conditions b^*_{-1}, b_{-1}, and m_{-1} and the set of sequences $\{R^*_t, R_t, \epsilon_t, e_t, P^N_t, w_t, \tau_t, C^N_t, C^T_t\}$.

From this problem we derive the optimization conditions which together with symmetry conditions and market clearing conditions can be used to find the laws of motion of the economy. These laws correspond to (1), (2), (27) with equality,

$$G\left(h^N_t, K^N\right) = c^N_t + \frac{\gamma}{2}\left(\pi^N_t - \bar{\pi}\right)^2 + \eta c^T_t \tag{28}$$

$$R_t = R^*(1 + \zeta_{t+1})\epsilon_{t+1} \tag{29}$$

$$-\frac{H_T(h_t^T)}{\tilde{U}_T(c_t^T, c_t^N)} = \frac{w_t}{1+\frac{\eta}{e_t}} \tag{30}$$

$$\tilde{U}_T(c_t^T, c_t^N) = \frac{\beta R_t}{\pi_{t+1}^T}\tilde{U}_T(c_{t+1}^T, c_{t+1}^N) \quad \text{where} \quad \pi_{t+1}^T = \epsilon_{t+1}\frac{\left(1+\frac{\eta}{e_{t+1}}\right)}{\left(1+\frac{\eta}{e_t}\right)} \tag{31}$$

$$\tilde{U}_N(c_t^T, c_t^N) = \frac{\beta R_t}{\pi_{t+1}^N}\tilde{U}_N(c_{t+1}^T, c_{t+1}^N) \tag{32}$$

$$\frac{\tilde{U}_N(c_{t+1}^T, c_{t+1}^N)(\pi_{t+1}^N - \bar{\pi}^N)\pi_{t+1}^N}{\tilde{U}_N(c_t^T, c_t^N)} = \frac{(\pi_t^N - \bar{\pi}^N)\pi_t^N}{\beta} + \frac{\mu\left(c_t^N + \eta c_t^T\right)}{\beta\gamma}\left(\frac{\mu-1}{\mu} - mc_t\right) \tag{33}$$

$$b_t^* - b_{t-1}^* = (R^*-1)b_{t-1}^* + R^* I_t + c_t^T - F\left(h_t^T, I_t, K^T\right) \tag{34}$$

$$F_T\left(h_t^T, K^T, I_t\right) = w_t(1+\zeta_t)R_t \tag{35}$$

$$mc_t = \frac{w_t e_t(1+\zeta_t)R_t}{G_N\left(h_t^N, K^N\right)} \tag{36}$$

$$F_I\left(h_t^T, K^T, I_t\right) = (1+\zeta_t)R^* \tag{37}$$

where $\lambda_t\zeta_t$ and λ_t are the Lagrange multipliers of the collateral constraint and the budget constraint. The latter multiplier evolves according to the asset pricing equation

$$\lambda_t = \beta R^*(1+\zeta_{t+1})\lambda_{t+1} \tag{38}$$

Equations (28)-(34) are basically equivalent to equations (8)-(14) in the simple model.[25] Therefore they have a similar interpretation. On the other hand equations (35)-(37) correspond to the optimal conditions that determine the household-firm unit demands for labor (for the production of the traded and non-traded good) and for the imported input.

A comparison between the laws of motion of the simple model and the augmented model reveal that the introduction of distribution costs, the collateral constraint, and the requirement of loans has some important consequences. On one hand distribution services affect the relative price of the traded good at the consumer level with respect to the nominal exchange rate. In the simple model this relative price was equal to one. In the augmented model this price is $1 + \frac{\eta}{e_t}$ which depends on the distribution costs parameter η. From (30) and (31) it is clear that through this price, distribution costs affect in equilibrium the optimal intratemporal decisions between labor and consumption of the traded good as well as the optimal intertemporal choices

[25] In the simple model $F_T\left(h_t^T\right) = w_t$

for consumption of the traded good. On the other hand distribution services generate an extra demand of non-traded goods as is captured by the last term, ηc_t^T, of the right hand side of (28). This extra demand also influences the dynamics of non-traded goods inflation as can be seen in (33).

The binding collateral generates an endogenous risk premium as reflected by the "modified" UIP condition in (29). In fact because of the constraint, the "effective" international nominal interest rate that domestic agents pay becomes $(1+\zeta_{t+1})R^*$. Thus raising external debt b_t^* not only requires the payment of interests $(R^* b_t^*)$ but also tightens the binding constraint $(\zeta_{t+1} > 0)$ generating an additional interest cost. Note also that in contrast to the simple model, in the augmented model we have assumed that the international interest rate (R^*) is constant and equal to $\frac{1}{\beta}$. Nevertheless in this context this typical assumption of the small open economy literature does not cause the unit-root problem.[26]

The requirement of loans to hire labor in tandem with the binding collateral constraint has an important effect on the labor demand decisions. By looking at the right hand sides of (35) and (36) it is possible to conclude the following. Ceteris paribus the necessity of short-term loans, the binding constraint, and the fact that in the short run $\zeta_t > 0$ imply that an increase in the "effective" interest rate $(1+\zeta_t)R_t$, will push the cost of hiring labor up. In response to this, the demand for labor for the production of the traded and non-traded goods will contract.

As mentioned before equation (37) corresponds to the optimal condition that determines the demand for the imported input. This condition equalizes the marginal product of this input to the effective cost of foreign working capital $(1+\zeta_t)R^*$ necessary to import it. As the constraint tightens and ζ_t goes up, the effective cost raises and the demand for the imported input decreases. Furthermore the purchases of this input influence the market equilibrium condition for the traded good as represented by equation (34). Since this equation describes the behavior of the current account then a decrease in the imported input can cause an improvement in the current account deficit. This improvement is almost immediate given the assumption that external short term loans to finance the intermediate input have to be repaid at the end of the period and not at the beginning of next period.

Finally although identical household-firm units will not trade capital in equilibrium, it is possible to derive the equilibrium value of the prices of capital. The equilibrium value of these asset prices correspond to

$$q_t^T = \frac{F_K + \left(\frac{R_t}{\epsilon_{t+1}}\right)^{-1} q_{t+1}^T}{(1-\phi\zeta_t)} \qquad \text{and} \qquad q_t^N = \frac{(\frac{mc_t}{e_t})G_K + \left(\frac{R_t}{\epsilon_{t+1}}\right)^{-1} q_{t+1}^N}{(1-\phi\zeta_t)} \tag{39}$$

where F_K and G_K are to the marginal products of capital in the production of the traded good and non-traded good respectively.

[26] Under this assumption and with the binding constraint, condition (38) becomes $\lambda_t = (1+\zeta_{t+1})\lambda_{t+1}$ which clearly does not present a unit root.

3.3 The Determinacy of Equilibrium Analysis

The definition of equilibrium in this set-up is the following.

Definition 2 *Given b^*_{-1}, R^*, K^N, K^T and the depreciation target $\bar{\epsilon}$, a symmetric perfect foresight equilibrium is defined as a set of sequences $\{c^T_t,\ c^N_t,\ \zeta_t,\ h^T_t,\ h^N_t,\ I_t,\ b^*_t,\ mc_t,\ e_t,\ q^T_t,\ q^N_t,\ w_t,\ \epsilon_t,\ \pi^N_t,\ R_t\}_{t=0}^{\infty}$ satisfying equations (1), (2), (27) with equality, (28)-(37) and (39).*

The methodology to pursue the determinacy of equilibrium analysis for this augmented model is the same as the one for the simple model. We log-linearize the system of equations that describe the equilibrium dynamics around the perfect-foresight steady state and characterize the dimension of the unstable subspace of the system comparing it to the number of non-predetermined variables. By log-linearizing we are following the same approach that Kiyotaki and Moore (1997) adopt to solve for an equilibrium of a model with a binding collateral constraint. By doing this we are precluding the possibility of exploring non-linear equilibrium dynamics.[27] Nevertheless the (log)linear approximation is what allows us to pursue the determinacy of equilibrium analysis for such a complex system.

In this set-up the steady state is calculated taking into account that the collateral constraint is binding. But by construction the shadow price of this constraint at the steady state is equal to zero. To see this use (38) in tandem with $\beta = \frac{1}{R^*}$ to obtain $\bar{\zeta} = 0$.[28] Nevertheless, in the short run the shadow price of the collateral constraint may vary as ζ_t changes. When $\zeta_t > 0$ is high, then the collateral constraint tightens.

It is not possible to derive analytical results for the log-linearized augmented model. Then we pursue some numerical simulations. To do so we need to choose some specific functional forms. For consumption and labor preferences we use

$$\tilde{U}(c^T_t, c^N_t) = \frac{\left[(\alpha)^{\frac{1}{\mathrm{a}}}\left(c^T_t\right)^{\frac{\mathrm{a}-1}{\mathrm{a}}} + (1-\alpha)^{\frac{1}{\mathrm{a}}}\left(c^N_t\right)^{\frac{\mathrm{a}-1}{\mathrm{a}}}\right]^{\left(\frac{\mathrm{a}}{\mathrm{a}-1}\right)(1-\sigma)} - 1}{1-\sigma}$$

$$H(h^T_t) + L(h^N_t) = -\frac{\varsigma}{1+\delta}\left[\left(h^T_t\right)^{1+\delta} + \left(h^N_t\right)^{1+\delta}\right]$$

where $\alpha \in (0,1)$, $\varsigma, \sigma, \mathrm{a} > 0$ and $\delta \geq 0$ whereas for technologies we utilize

$$F\left(h^T_t, K^T, I_t\right) = \left\{\varpi\left[\vartheta_1 (h^T_t)^{\theta^T}(K^T)^{1-\theta^T}\right]^{\frac{\chi-1}{\chi}} + (1-\varpi)\left[\vartheta_2 I_t\right]^{\frac{\chi-1}{\chi}}\right\}^{\frac{\chi}{\chi-1}}$$

[27] To pursue a determinacy of equilibrium analysis to the augmented non-linear model is a very difficult task. In fact most of the aforementioned works that include a collateral constraint and that simulate equilibrium dynamics for the non-linear system, do not characterize the equilibrium. They assume that the equilibrium that is found is the unique and relevant equilibrium whose properties should be studied.

[28] Since $\bar{\zeta} = 0$ then to be able to log-linearize the system of equations of Definition 2, we define the new variable $\zeta^n_t \equiv 1 + \zeta_t$ whose steady state value is $\bar{\zeta}^n = 1$.

$$G\left(h_t^N, K^N\right) = \left(h_t^N\right)^{\theta^N} \left(K^N\right)^{1-\theta^N}$$

where $\theta^N, \theta^T, \varpi \in (0,1)$ and $\vartheta_1, \vartheta_2, \chi > 0$.

In order to assign values to the parameters of the model we use mainly the calibration of Christiano et al. (2004). The only values that are not taken from this work are the intratemporal elasticity of substitution (a), the parameter related to distribution services (η), the parameter that governs the degree of price stickiness (γ), and the parameter associated with the degree of imperfect competition (μ).[29] We do not pick any particular value for the interest rate response coefficient to currency depreciation (ρ_ϵ) since we will study how this parameter affects the determinacy of equilibrium. We choose values for "a" and η that are in line with similar values used in the distribution services literature.[30] Since there are no robust estimates of a New-Keynesian Phillips curve for emerging economies we choose values for γ and μ that are consistent with the values used in the closed economy literature about nominal price rigidities.[31] Table 2 summarizes the parametrization.

Table 2

R^*	β	$\bar{R}$	γ	μ	η	ϕ	α	a	σ
1.06	0.943	1.16	17.5	6	0.85	0.185	0.7	0.4	2

ς	δ	ϖ	ϑ_1	ϑ_2	θ^T	χ	θ^N	K^T	K^N
4.59	5	0.6	1.4	3.5	0.5	0.7	0.64	1	2

Using this parametrization we can study how the determinacy of equilibrium varies with respect to the response coefficient to depreciation (ρ_ϵ) of the policy rule (2) and other structural parameters. As an illustrative case we focus on the experiment of characterizing the equilibrium while we vary the degree of responsiveness to current currency depreciation (ρ_ϵ) and the intratemporal elasticity of substitution (a) keeping the rest constant. The results are presented in Figure 1. In this Figure a cross "x" denotes combinations of these parameters under which the policy induces multiple cyclical equilibria whose degree of indeterminacy is of order one. On the other hand, a dot "." represents parameter combinations under which the policy induces multiple cyclical equilibria whose degree of indeterminacy is of order two.[32]

From Figure 1 we can derive the following conclusions. In the augmented set-up a policy that responds to current currency depreciation by raising ($\rho_\epsilon > 0$ with $\rho_\epsilon \neq 1$) or lowering ($\rho_\epsilon < 0$ with $\rho_\epsilon \neq -1$) the nominal

[29] Note that target nominal depreciation rate, $\bar{\epsilon}$ can be found evaluating (29) at the steady state. That is, $\bar{\epsilon} = \frac{\bar{R}}{R^*}$. The value of $\bar{R}$ that we take is close to the one in Christiano et al. (2004).

[30] See Burnstein et al. (2005a,b).

[31] See Schmitt-Grohé and Uribe (2004) among others.

[32] The presence of cyclical equilibria is associated with the existence of non-explosive complex roots in the multidimensional system.

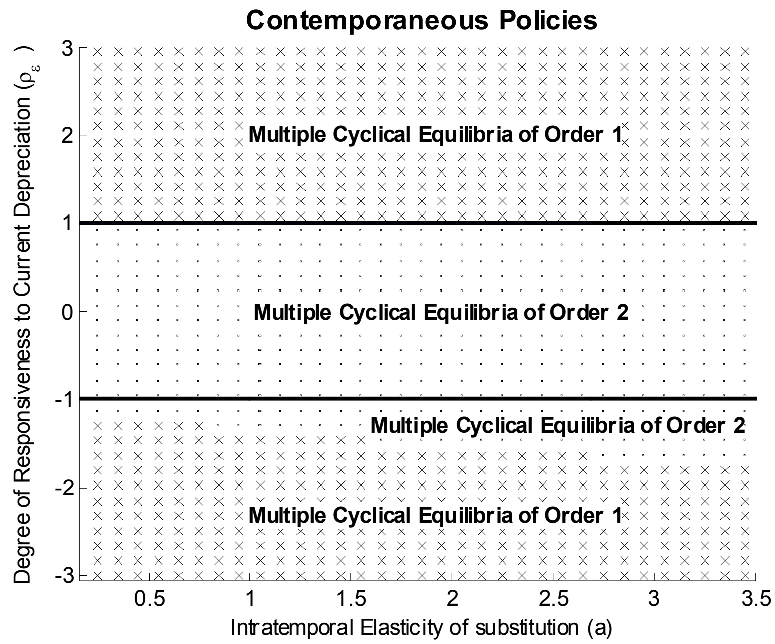

Figure 1: This Figure shows the characterization of the equilibrium for interest rate policies $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ varying the degree of responsiveness to currency depreciation (ρ_ϵ) and the intratemporal elasticity of substitution (a). It is assumed that $\rho_\epsilon \neq -1, 0, 1$. A cross "x" denotes parameter combinations under which the policy induces multiple cyclical equilibria whose degree of indeterminacy is of order one. A dot "." represents parameter combinations under which the policy induces multiple cyclical equilibria whose degree of indeterminacy is of order two.

interest rate can induce multiple equilibria regardless of the intratemporal elasticity of substitution "a". This suggests that the results in the augmented set-up are similar to the ones in the simple set-up. Nevertheless there is an important distinction. In the augmented model because of the binding collateral constraint, the previously mentioned policy leads to self-fulfilling "cycles" or, equivalently, to multiple "cyclical" equilibria. This should not be a surprise. It is just a consequence of two mechanisms working together. On one hand from the results in the simple model we have that this policy can induce self-fulfilling non-cyclical fluctuations. On the other hand from Kiyotaki and Moore (1997) we know that the introduction of a binding collateral constraint can cause "credit cycles". Hence the combination of the two mechanisms can lead to "self-fulfilling cyclical equilibria."

Experiments with respect to other structural parameters different from the intratemporal elasticity of substitution (a) lead to similar results.[33] Based on this we state the following proposition.

Proposition 2 *Under a crisis when the collateral constraint is binding if the government follows a contemporaneous interest rate policy that responds to currency depreciation ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ with $\rho_\epsilon \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$), then there exists a continuum of "cyclical" perfect foresight equilibria in which the*

[33] The results are available from the author upon request.

sequences $\{\hat{c}_t^T,\ \hat{c}_t^N,\ \hat{\zeta}_t,\ \hat{h}_t^T,\ \hat{h}_t^N,\ \hat{I}_t,\ \hat{b}_t^*,\ \hat{mc}_t,\ \hat{e}_t,\ \hat{q}_t^T,\ \hat{q}_t^N,\ \hat{w}_t,\ \hat{\epsilon}_t,\ \hat{\pi}_t^N,\ \hat{R}_t\}_{t=0}^{\infty}$ *converge asymptotically to the steady state.*

Are these results specific to the type of interest rate policy we are considering? To answer this question we can pursue a sensitivity analysis considering other rules. In particular we can characterize the equilibrium in the economy under policies that respond exclusively to either future depreciation (forward-looking policies $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$) or past depreciation (backward-looking policies $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$). The numerical results from this analysis are presented in the Appendix. They imply that the answer to the previous question is no: as long as the interest rate policy responds to the depreciation rate, then the policy can induce multiple equilibria. Forward-looking policies $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ with $\rho_\epsilon \neq 0$ always induce multiple cyclical equilibria when either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$. Except for the presence of cycles these results still agree with the ones from the simple model.

On the other hand, for backward-looking policies $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ with $\rho_\epsilon \neq 0$, the coefficient of response to past depreciation ρ_ϵ plays an important role in the characterization of the equilibrium: timid rules satisfying $|\rho_\epsilon| < 1$ always induce multiple equilibria; aggressive rules with $|\rho_\epsilon| > 1$ can guarantee a unique equilibrium. Nevertheless being aggressive with respect to past depreciation ($|\rho_\epsilon| > 1$) is not a sufficient condition to guarantee a unique equilibrium. It is only a necessary condition. In other words, backward-looking rules can still induce aggregate instability by generating self-fulfilling cyclical fluctuations. These results as well as the results for a contemporaneous policy $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$ from Proposition 2 are summarized in Table 1 in the columns labeled as "The Augmented Model."

From this Table we can observe that, to some extent, the possibility of multiple equilibria arises regardless of whether the nominal interest rate is raised or lowered in response to either current, future or past currency depreciation. In this sense our results do not provide any support to any of the policy recommendations that were part of the debate about the right interest rate policy in the aftermath of the Asian crisis. They just point out some of the negative consequences of using the nominal interest rate as an exclusive instrument to respond to past, current or future currency depreciation.

It is possible to argue that in the aftermath of a crisis governments may maneuver the nominal interest rate in response not only to currency depreciation but also to inflation. This poses the question of whether an interest rate policy that reacts to both the CPI-inflation and the depreciation rate will induce aggregate instability in the economy by generating multiple equilibria. The answer to this question is affirmative making our previous results stronger. It is the reaction to currency depreciation what opens the possibility of multiple equilibria. To see this we can study a rule that, besides reacting to current currency depreciation, responds aggressively and positively to past CPI-inflation. That is, in log-linearized terms

$$\hat{R}_t = \rho_\pi \hat{\pi}_{t-1}^{cpi} + \rho_\epsilon \hat{\epsilon}_t \qquad \text{with} \qquad \rho_\pi > 1,\ \rho_\epsilon \neq 0 \text{ and either } |\rho_\epsilon| > 1 \text{ or } |\rho_\epsilon| < 1$$

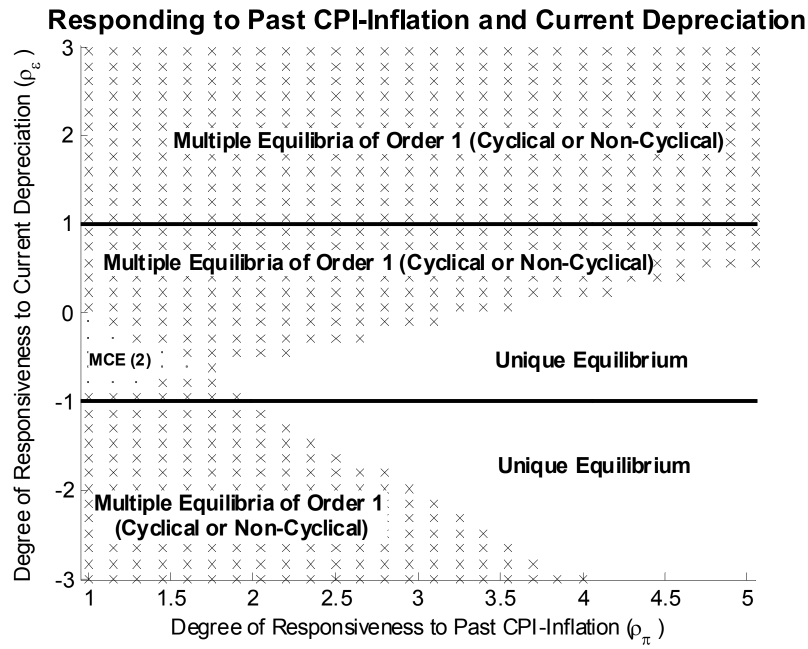

Figure 2: This Figure shows the characterization of the equilibrium for the rule $\hat{R}_t = \rho_\pi \hat{\pi}^{cpi}_{t-1} + \rho_\epsilon \hat{\epsilon}_t$ varying the degrees of responsiveness to past CPI-inflation (ρ_π) and to current currency depreciation (ρ_ϵ). It is assumed that $\rho_\epsilon \neq -1, 0, 1$. A cross "x" denotes parameter combinations associated with multiple equilibria whose degree of indeterminacy is of order one. These equilibria can be cyclical or non-cyclical. A dot "." represents parameter combinations associated with multiple equilibria whose degree of indeterminacy is of order two. These equilibria are cyclical and we name these combinations as "MCE(2)". The white regions represent parameter combinations under which there exists a unique equilibrium.

The reason for analyzing this policy is that the literature of interest rate rules claims that an aggressive backward-looking rule with respect to inflation is more prone to guarantee a unique local equilibrium than forward-looking and contemporaneous rules.[34] Hence by analyzing such a rule we can isolate and unveil the mechanism that opens the possibility of indeterminacy. As argued before this mechanism is associated with the response to currency depreciation.

The results of the analysis are shown in Figure 2. We study how variations of the degrees of responsiveness to past CPI-inflation and current depreciation, ρ_π and ρ_ϵ, affect the characterization of the equilibrium. From this figure it is clear that even backward-looking rules that respond aggressively with respect to the CPI-inflation can induce multiple equilibria. In fact under the celebrated "Taylor coefficient" $\rho_\pi = 1.5$, any rule will lead to real indeterminacy as long as the response to currency depreciation is positive or negative. Interestingly if the rule is positively aggressive with respect to current depreciation ($\rho_\epsilon > 1$) then it would be necessary to have an excessively aggressive rule with respect to inflation ($\rho_\pi > 5$) to avoid the possibility of self-fulfilling equilibria.

[34] See Woodford (2003).

In this section we have pointed out some possible perils of responding to currency depreciation by raising or lowering the nominal interest rate in the aftermath of a crisis. But there is still a relevant question that has not been answered: if we believe that the governments of the Asian economies followed these interest rate policies then is it possible to support the stylized facts of the aftermath of the crisis as one of these self-fulfilling equilibria? An affirmative answer to this question will make our previous theoretical results more credible. The next section provides an answer to this question.

4 Constructing a Self-fulfilling Cyclical Equilibrium

In this Section we use the augmented model in tandem with an interest rate policy that responds to currency depreciation in order to construct a self-fulfilling cyclical equilibrium that replicates some of the stylized facts of the aftermath of emerging crises. In particular we construct an equilibrium characterized by the self-validation of people's expectations about currency depreciation and by some of the stylized facts of the "Sudden Stop" phenomenon.

We assume the government follows an interest rate policy $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_t$, that responds aggressively ($|\rho_\epsilon| > 1$) to current depreciation, $\hat{\epsilon}_t$.[35] This policy captures the immediate reaction of the government to present conditions about currency depreciation. Unfortunately in the empirical literature that emerged after the Asian crisis there are no robust estimates for the parameter ρ_ϵ. As we discussed before this is one of the problems and drawbacks of the literature. For illustrative purposes we set $\rho_\epsilon = 1.5$ which implicitly reflects that government tends to increase proportionally the nominal interest rate more than the increase in currency depreciation. From Figure 1 we know that varying ρ_ϵ will not preclude the possibility of multiple equilibria although it changes the degree of indeterminacy. Since we want to construct a self-fulfilling cyclical equilibrium in which a sunspot affects exclusively people's expectations about one variable of the economy such as currency depreciation, then we need a value of ρ_ϵ that satisfies $|\rho_\epsilon| > 1$. This implies a degree of indeterminacy of order one.[36] By choosing $\rho_\epsilon = 1.5$ and keeping the rest constant, we achieve this goal.[37] As long as $\rho_\epsilon > 1$, increasing or reducing ρ_ϵ will not change the qualitative results that we will present and that capture some of the stylized facts of the "Sudden Stops."

It is important to clarify that in the construction of our self-fulfilling equilibrium we are assuming exogenously the occurrence of a crisis. That is, the binding collateral constraint is exogenously imposed at time $t = 0$ as in Christiano et al. (2004). In this sense we are only interested in studying what happens in the economy at and in the aftermath of the crisis. In what follows, therefore, we concentrate exclusively in the

[35] It is also possible to construct self-fulfilling equilibria with the forward-looking and backward-looking rules that in principle replicate most of the stylized facts.

[36] If the degree of indeterminacy were 2, then we would have an extra degree of freedom. We could assume that the sunspot affects the expectations of an extra variable different from the depreciation rate. In this sense we are being conservative.

[37] That is, in this case the dynamic log-linearized system that describes the economy has complex and non-explosive eigenvalues and the number of non-predetermined variables exceeds the number of explosive eigenvalues by one.

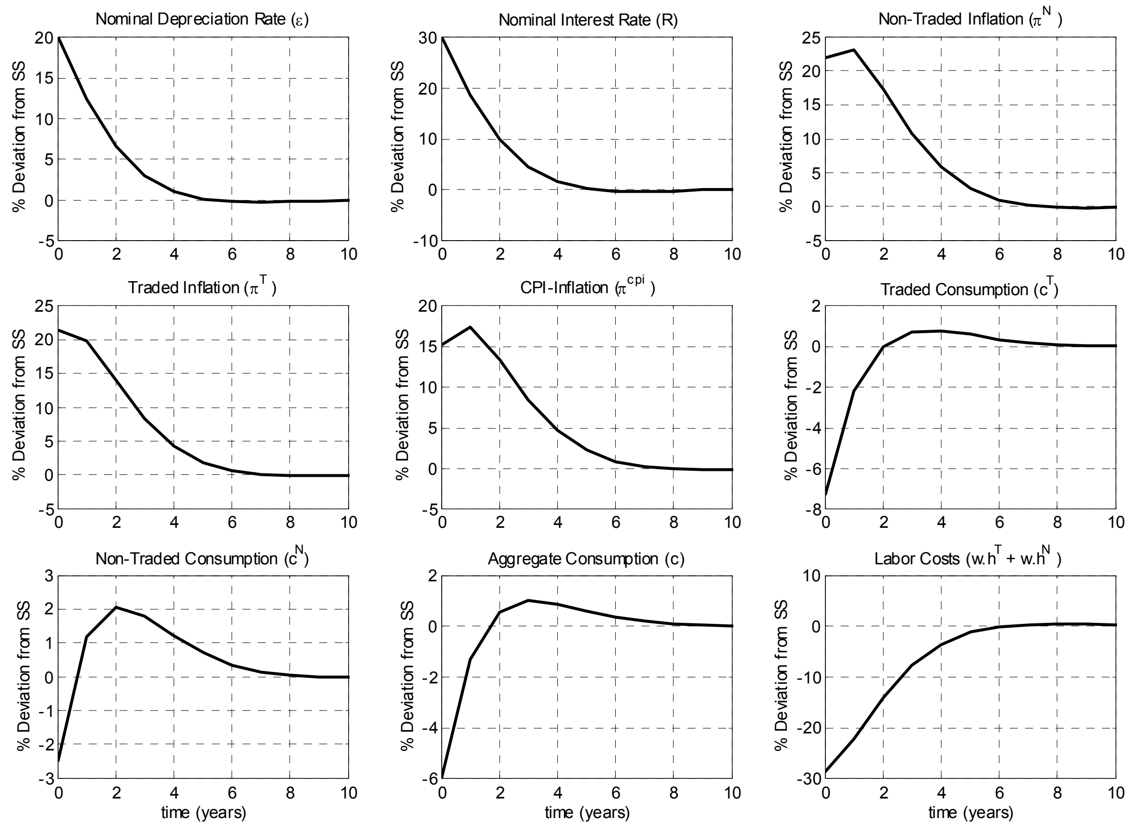

Figure 3: Impulse Responses of a self-fulfilling equilibrium when at time $t = 0$ people expect a higher nominal depreciation rate $\hat{\epsilon}_0 = 20\%$. The figure shows that this equilibrium replicates a decline of consumption of traded and non-traded goods that is present in a "Sudden Stop". It also replicates a relatively moderate increase in CPI-inflation, a higher depreciation rate and a higher nominal interest rate. All the variables are measured as percentage deviations from the steady state.

equilibrium dynamics of the economy at and after $t = 0$.

Imagine that when the crisis hits the economy and the collateral constraint binds at time $t = 0$, people develop expectations, in response to a sunspot, of a 20% higher nominal depreciation. By the determinacy of equilibrium analysis we know that these expectations will be self-validated. More importantly these expectations will cause a chain of events that are described by the impulse response functions of some of the macroeconomic variables presented in Figures 3 and 4. In these figures all the variables but the multiplier of the collateral constraint are measured as percentage deviations from the steady state. A quick inspection reveals that at time $t = 0$ and for some subsequent periods, the self-fulfilling equilibrium captures the following stylized facts of the "Sudden Stops": a decline in the aggregate demand (consumptions of traded and non-traded goods and aggregate consumption), a collapse in the domestic production (traded output and

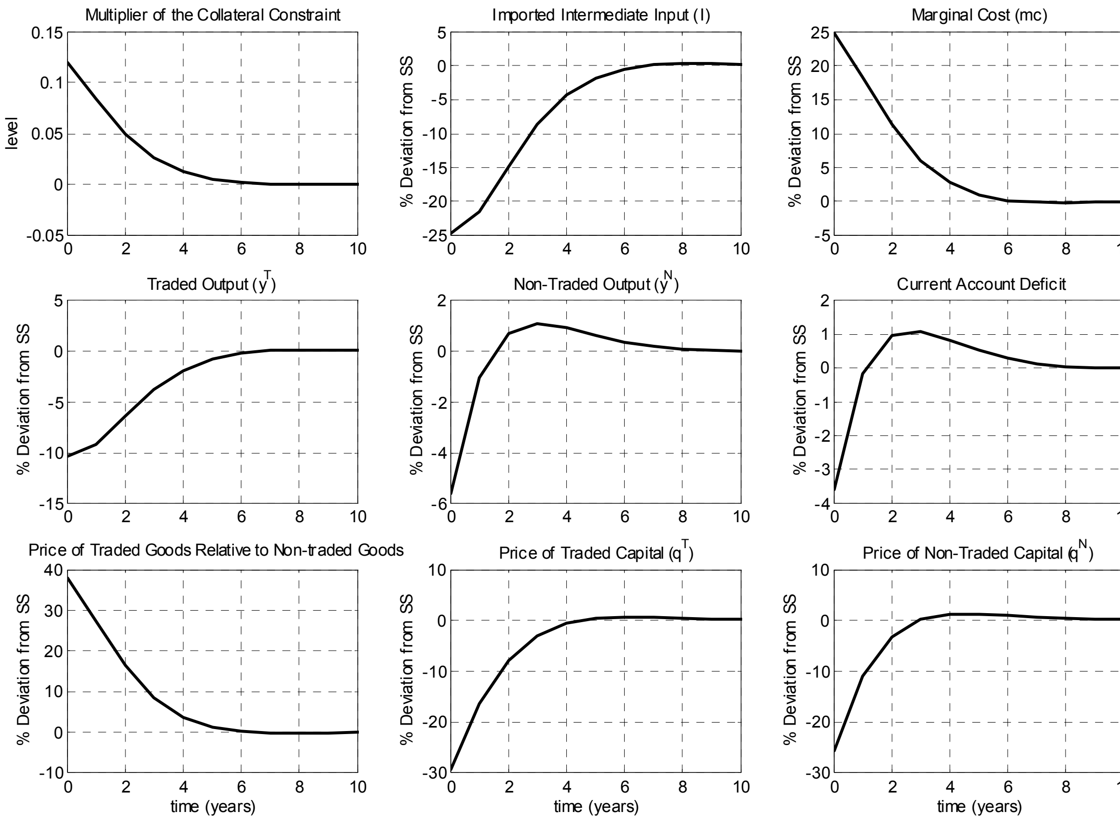

Figure 4: Impulse Responses of a self-fulfilling equilibrium when at time $t = 0$ people expect a higher nominal depreciation rate $\hat{\epsilon}_0 = 20\%$. The figure shows that this equilibrium replicates some of the "Sudden Stops" stylized facts: a collapse in the domestic production (of the traded and non-traded good), a collapse in asset prices (prices of capital), a sharp correction in the price of traded goods relative to non-traded goods and an improvement in the current account deficit. All the variables are measured as percentage deviations from the steady state.

non-traded output), a collapse in asset prices (prices of traded and non-traded capital), a sharp correction in the price of traded goods relative to non-traded goods, an improvement in the current account deficit, a moderate higher CPI-inflation, a higher currency depreciation rate and a higher nominal interest rate. After some periods the cycles are quickly dampened and the economy converges to the steady state.

To provide an explanation of these results is not an easy task given the interaction and interdependence of all the variables. Nevertheless we can give the following logical arguments. The self-validated increase in the nominal depreciation rate implies an increase in the nominal exchange rate and, by the rule, leads to an increase in the nominal interest rate at $t = 0$. Since the rule is aggressive, the nominal interest rate rises by more than both the expected non-traded good inflation rate and the expected traded good inflation rate at time $t = 1$.[38] Hence the real interest rate at $t = 0$ measured in terms of either the expected traded inflation

[38] This is probably the case because there is sluggish price adjustment for the price of the non-traded good that in turn affects

or the non-traded inflation at $t = 1$ goes up. Provided that this induces an intertemporal substitution effect in consumption that more than offsets any intratemporal substitution effect, then consumption of the traded good and consumption of the non-traded good decline at $t = 0$. This can be inferred from (31) and (32). As a result of this, aggregate consumption also decreases implying, to some extent, that the model is able to capture the initial decline in aggregate demand present in the "Sudden Stops."

Since the real value of capital as a collateral is expressed in terms of foreign currency then the previously mentioned increase of the nominal exchange rate at $t = 0$ reduces this value. Hence, by the collateral constraint, less international loans will be available to buy the intermediate imported input. As long as the costs of hiring labor rise, due to a higher interest rate, then a shortage in international loans will be translated into a reduction in the demand for the imported input at $t = 0$. This can be deduced from (27) taking into account that b^*_{t-1} is a predetermined variable.

At the same time, a higher interest rate in response to currency depreciation will also push up the costs of loans to hire labor utilized in the production of traded goods. This implies that the household-firm unit will cut back labor in the production of the traded good which in tandem with the reduction in the imported input leads to a decrease in output at $t = 0$.[39] On the other hand the supply of the non-traded good is demand determined. This supply satisfies consumption of non-traded goods and distribution services for traded goods. Consequently the previously mentioned decrease in demand of both goods causes a decrease in non-traded output (labor) at $t = 0$. Thus the model is able to capture the decline in output of both non-traded and traded goods. And provided that the decrease in traded output is smaller than the contractions in consumption of the traded good and the imports of the intermediate input then an improvement in the current account is also possible as can be inferred from (34).

As the collateral constraint tightens and the nominal interest rate rises then the "effective" nominal interest rate $(1 + \zeta_t)R_t$ increases. This pushes marginal costs of producing the non-traded good up forcing the household-firm unit to raise the price of this good; which in turn leads to a higher non-traded goods inflation rate as can be deduced from (36) and (33). To some extent price-stickiness will guarantee that the increase in non-traded inflation is smaller than the increase the depreciation rate. This together with distribution services has two important consequences. First as a consequence of large depreciations there will be a sharp correction in the price of the traded good relative to the price of non-traded good at the consumer level $\left(\frac{P_t^T}{P_t^N}\right)$. Second in the short run the consumer-price-index (CPI) inflation will be below the depreciation rate. Both facts agree with some empirical regularities present in the aftermath of a crisis.

The real value of a unit of capital for traded output (respectively for non-traded output) in terms of foreign currency is determined by the net present value of the flows of the marginal product of capital in

the price of the traded good through the existence of distribution services.

[39] This can be deduced from (35) and the production technology of the traded good. Labor costs increase not only because the nominal interest rate increases but also because the collateral constraint tightens. That is there is an increase in the "effective" nominal interest rate $(1 + \zeta_t)R_t$.

the production of the traded good (respectively non-traded good). This can be seen by iterating forward equations (39). Then provided that capital is constant in the analysis, the reduction of labor and the intermediate input will cause a decline in the marginal product of capital. This in turn will affect negatively the real value of capital for traded output. A similar mechanism will lead to a decrease in the real value of capital for non-traded output. Thus asset prices fall capturing another stylized fact of the "Sudden Stops."

5 Concluding Remarks

In this paper we show that in the aftermath of a crisis, a government that changes the nominal interest rate in response to currency depreciation can induce aggregate instability in the economy by generating self-fulfilling endogenous cycles. In this sense a policy that originally attempts to stabilize the nominal exchange rate and the whole economy leads to complete opposite effects.

We also show that if a government raises the interest rate proportionally more than an increase in currency depreciation then it induces self-fulfilling cyclical equilibria that are able to replicate some of the empirical regularities of emerging market crises. In fact we construct an equilibrium based on the self-validation of people's expectations about currency depreciation that is able to replicate the following "Sudden Stop" stylized facts: a decline in domestic production and aggregate demand, a moderate higher CPI-inflation, a significantly larger currency depreciation, a higher nominal interest rate, a collapse in asset prices, a sharp correction in the price of traded goods relative to non-traded goods and an improvement in the current account deficit.

The implications of our results are interesting. Previous works have emphasized that this interest rate policy can cause fiscal and output costs. Our results suggest that this policy can be also costly to the extent that it can induce macroeconomic instability in the economy by opening the possibility of "sunspot" equilibria. These equilibria that are not driven by fundamentals can be associated with a large degree of volatility for some macroeconomic aggregates such as consumption. Provided that agents are risk averse then volatile consumption will cause a decrease in welfare.

Our results also provide a possible explanation of why the empirical literature has not been able to disentangle the relationship between interest rates and the nominal exchange (depreciation) rate in the aftermath of a crisis. This literature has tried to control for the variables that influence the nominal exchange rate. But our results suggest that there can be potential influences that may depend on "sunspots" which in turn can induce self-fulfilling cycles in the nominal exchange rate (or the nominal depreciation rate) as well as in other variables. Clearly these influences do not depend on fundamentals and their effect is something that the empirical literature should take into account.

An issue that we have not addressed is whether the (self-fulfilling) sunspot equilibria are learnable by the agents of the economy. It was implicitly assumed that agents could coordinate their actions on any particular equilibrium. We could relax this assumption and use the Expectational Stability concept developed in Evans and Honkapojha (2001) to pursue a learnability analysis. We leave this for future research.

A Appendix

This Appendix has two parts. The first part includes material that supports the analysis for the simple model of Section 2. The second part includes the simulations of the determinacy of equilibrium analysis for forward-looking and backward-looking rules for the augmented model in Section 3.

A.1 The Simple Model

A.1.1 The First Order Conditions of the Household-Firm Unit Problem in the Simple Model

The representative household-firm unit chooses the set of sequences $\{c_t^T,\, c_t^N,\, h_t^T,\, h_t^N,\, \check{h}_t^T,\, \tilde{h}_t^N,\, \tilde{P}_t^N,\, b_t^*,\, b_t,\, m_t\}_{t=0}^{\infty}$ in order to maximize (3) subject to (4), (5), (6) and (7), given the initial condition n_{-1} and the set of sequences $\{R_t^*,\, R_t,\, \epsilon_t,\, e_t,\, P_t^N,\, w_t,\, \tau_t,\, C_t^N\}$. The first order conditions correspond to (5) and (7) with equality and

$$U_T(c_t^T) = \lambda_t \tag{40}$$

$$\frac{U_T(c_t^T)}{V_N(c_t^N)} = e_t \tag{41}$$

$$-\frac{H_T(h_t^T)}{U_T(c_t^T)} = w_t \tag{42}$$

$$-\frac{L_N(h_t^N)}{V_N(c_t^N)} = w_t e_t \tag{43}$$

$$1 = \frac{w_t}{F_T\left(\check{h}_t^T\right)} \tag{44}$$

$$mc_t = \frac{w_t e_t}{G_N\left(\tilde{h}_t^N\right)} \tag{45}$$

$$\begin{aligned} 0 \;=\; & \frac{\lambda_t C_t^N}{e_t} d\left(\frac{\tilde{P}_t^N}{P_t^N}\right) - \gamma\frac{\lambda_t}{e_t}\left(\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} - \bar{\pi}^N\right)\frac{\tilde{P}_t^N}{\tilde{P}_{t-1}^N} + \left(\frac{\tilde{P}_t^N}{P_t^N} - mc_t\right)\frac{\lambda_t C_t^N}{e_t} d'\left(\frac{\tilde{P}_t^N}{P_t^N}\right) \\ & +\beta\gamma\frac{\lambda_{t+1}}{e_{t+1}}\left(\frac{\tilde{P}_{t+1}^N}{\tilde{P}_t^N} - \bar{\pi}^N\right)\frac{\tilde{P}_{t+1}^N}{\tilde{P}_t^N} \end{aligned} \tag{46}$$

$$J_m(m_t) = U_T(c_t^T)\left(\frac{R_t - 1}{R_t}\right) \tag{47}$$

$$\lambda_t = \beta R_t^* \lambda_{t+1} \tag{48}$$

$$\lambda_t = \frac{\beta R_t}{\epsilon_{t+1}} \lambda_{t+1} \tag{49}$$

where $\frac{mc_t \lambda_t}{e_t}$ and λ_t correspond to the Lagrange multipliers of (4) and (5) respectively.

We will focus on a symmetric equilibrium in which all the monopolistic producers of sticky-price non-traded goods pick the same price. Hence $\tilde{P}_t^N = P_t^N$. Since all the monopolists face the same wage rate, W_t, and the same production function, $G\left(h_t^N\right)$, then they will demand the same amount of labor $\tilde{h}_t^N = h_t^N$. In equilibrium the money market, the domestic bond market, the labor markets, the non-traded goods market and the traded good market clear. Therefore

$$m_t = m_t^g \tag{50}$$

$$b_t = b_t^g \tag{51}$$

$$h_t^T = \check{h}_t^T \tag{52}$$

$$h_t^N = \tilde{h}_t^N \tag{53}$$

$$G\left(h_t^N\right) = c_t^N + \frac{\gamma}{2}\left(\pi_t^N - \bar{\pi}^N\right)^2 \tag{54}$$

and

$$b_t^* = R_{t-1}^* b_{t-1}^* + c_t^T - F\left(h_t^T\right) \tag{55}$$

Combining (48) and (49) yields the uncovered interest parity condition (9). From (42) and (44) we can derive equation (10). Using conditions (40) and (48) we obtain the Euler equation for consumption of the traded good that corresponds to (11). Utilizing (1), (40), (41), and (49) we derive the Euler equation (12) for consumption of the non-traded good. And finally using the notion of a symmetric equilibrium, conditions (4), (41), (43), (45), (53), (46) and the definitions $\pi_t^N = P_t^N / P_{t-1}^N$, $d(1) = 1$ and $d'(1) = -\mu$ we can derive the augmented Phillips curve described by equation (13).

A.1.2 Characterization of the Steady State in the Simple Model

We use $\beta R^* = 1$, (1), (8)-(15), and the condition that at the steady state $x_t = \bar{x}$ for all the variables to derive

$$\bar{\pi}^N = \bar{\epsilon} \qquad \beta \bar{R} = \bar{\epsilon} \qquad 1 = \beta \bar{R}^* \qquad \bar{R}^* = R^*$$

$$\left(\frac{\mu - 1}{\mu}\right) V_N(G(\bar{h}^N)) = -\frac{L_N(\bar{h}^N)}{G_N\left(\bar{h}^N\right)}$$

$$U_T\left((1 - 1/\beta)\,\bar{b}^* + F\left(\bar{h}^T\right)\right) = -\frac{H_T(\bar{h}^T)}{F_T\left(\bar{h}^T\right)}$$

$$\bar{c}^N = G\left(\bar{h}^N\right) \qquad \bar{c}^T = (1 - 1/\beta)\,\bar{b}^* + F\left(\bar{h}^T\right)$$

Then it is simple to prove that under some assumptions that include Assumptions 1, 2, and 3, and given $\mu > 1$, $\beta \in (0,1)$ and $\bar{\epsilon} > 1$, there exists a steady state $\{\bar{c}^N, \bar{c}^T, \bar{h}^T, \bar{h}^N, \bar{\pi}^N, \ \bar{R}, \bar{R}^*\}$ for the economy that provided a particular $\bar{b}^*$ satisfies these equations with $\bar{c}^T, \bar{c}^N, \bar{h}^T, \bar{h}^N > 0$ and $\bar{\pi}^N, \bar{R}, \bar{R}^* > 1$. In particular to guarantee that there exist $\bar{c}^T, \bar{c}^N, \bar{h}^T, \bar{h}^N > 0$ we need some Inada-type assumptions such as $U_T(0) = V_N(0) = \infty$, $U_T(\infty) = V_N(\infty) = 0$, $F_T(0) = G_N(0) = \infty$, and $F_T(\infty) = G_N(\infty) = 0$.[40]

A.1.3 Forward-Looking and Backward-Looking Rules in the Simple Model

In order to pursue the determinacy of equilibrium analysis for forward-looking rules, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$, we use this rule and equations (17)-(21) to obtain

$$\begin{pmatrix} \hat{b}^*_t \\ \hat{c}^T_{t+1} \\ \hat{\pi}^N_{t+1} \\ \hat{c}^N_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1+\psi}{\beta} & \kappa & 0 & 0 \\ \frac{\psi(1+\psi)\xi^T}{\beta} & \left(1+\psi\kappa\xi^T\right) & 0 & 0 \\ 0 & 0 & \frac{1}{\beta} & -\varphi \\ \frac{\rho_\epsilon\psi(1+\psi)\xi^N}{\beta(\rho_\epsilon-1)} & \frac{\rho_\epsilon\psi\kappa\xi^N}{(\rho_\epsilon-1)} & -\frac{\xi^N}{\beta} & \left(1+\varphi\xi^N\right) \end{pmatrix}}_{J^f} \begin{pmatrix} \hat{b}^*_{t-1} \\ \hat{c}^T_t \\ \hat{\pi}^N_t \\ \hat{c}^N_t \end{pmatrix} \tag{56}$$

Then the determinacy of equilibrium analysis delivers the results stated in the following Proposition.

Proposition 3 *If the government follows a forward-looking interest rate rule such as $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ with $\rho_\epsilon \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$, then there exists a continuum of perfect foresight equilibria in which the sequences $\{\hat{b}^*_t, \ \hat{c}^T_t, \hat{\pi}^N_t, \ \hat{c}^N_t\}_{t=0}^{\infty}$ converge asymptotically to the steady state. The degree of indeterminacy is of order 1.*[41]

Proof. The eigenvalues of the matrix J^f in (56) correspond to the roots of the characteristic polynomial $\mathcal{P}^f(v) = \left|J^f - vI\right| = 0$ whose definition is provided in (59). By Lemma 4 we know that the characteristic polynomial $\mathcal{P}^f(v) = 0$ has real roots satisfying $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$ and $|v_4| > 1$. Therefore $\mathcal{P}^f(v) = 0$ has only two explosive roots, which means that the matrix J^f in (56) has two explosive eigenvalues. Given that there are three non-predetermined variables namely $\hat{c}^T_t$, $\hat{\pi}^N_t$ and $\hat{c}^N_t$, then the number of non-predetermined variables is greater than the number of explosive roots. Applying the results of Blanchard and Kahn (1980) it follows that there exists an infinite number of perfect foresight equilibria converging to the steady state. In addition the difference between the number of non-predetermined variables and explosive roots implies that the degree of indeterminacy is of order 1. ■

To derive the results for backward-looking policies we use the log-linearized version of the rule, $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ together with equations (20)-(17) to obtain the log-linearized system

[40] Details are available from the author upon request.

[41] The degree of indeterminacy is defined as the difference between the number of non-predetermined variables and the dimension of the unstable subspace of the log-linearized system.

$$
\begin{pmatrix} \hat{R}_{t+1} \\ \hat{\epsilon}_{t+1} \\ \hat{b}_t^* \\ \hat{c}_{t+1}^T \\ \hat{\pi}_{t+1}^N \\ \hat{c}_{t+1}^N \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & \rho_\epsilon & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{\psi(1+\psi)}{\beta} & -\psi\kappa & 0 & 0 \\ 0 & 0 & \frac{1+\psi}{\beta} & \kappa & 0 & 0 \\ 0 & 0 & \frac{\psi(1+\psi)\xi^T}{\beta} & \left(1+\psi\kappa\xi^T\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\beta} & -\varphi \\ \xi^N & 0 & 0 & 0 & -\frac{\xi^N}{\beta} & \left(1+\varphi\xi^N\right) \end{pmatrix}}_{J^b} \begin{pmatrix} \hat{R}_t \\ \hat{\epsilon}_t \\ \hat{b}_{t-1}^* \\ \hat{c}_t^T \\ \hat{\pi}_t^N \\ \hat{c}_t^N \end{pmatrix} \tag{57}
$$

We use this system to pursue the determinacy of equilibrium analysis and derive the results in the following proposition.

Proposition 4 *Assume the government follows a backward-looking rule such as $\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ with $\rho_\epsilon \neq 0$.*

> ***a)*** *if $|\rho_\epsilon| > 1$ then there exists a unique perfect foresight equilibria in which the sequences $\{\hat{R}_t,\ \hat{\epsilon}_t,\ \hat{b}_t^*,\ \hat{c}_t^T,\ \hat{\pi}_t^N,\ \hat{c}_t^N\}_{t=0}^{\infty}$ converge to the steady state.*
>
> ***b)*** *if $|\rho_\epsilon| < 1$ then there exists a continuum of perfect foresight equilibria in which the sequences $\{\hat{R}_t,\ \hat{\epsilon}_t,\ \hat{b}_t^*,\ \hat{c}_t^T,\ \hat{\pi}_t^N,\ \hat{c}_t^N\}_{t=0}^{\infty}$ converge asymptotically to the steady state. In addition the degree of indeterminacy is of order 2.*

Proof. The eigenvalues of the matrix J^b in (57) correspond to the roots of the characteristic polynomial $\mathcal{P}^b(v) = \left|J^b - vI\right| = 0$. Using the definition of J^b in (57) this polynomial can be written as

$$
\mathcal{P}^b(v) = \left(v^2 - \rho_\epsilon\right) \mathcal{P}^f(v) = 0 \tag{58}
$$

where $\mathcal{P}^f(v)$ is defined in (59). By Lemma 4 we know that the characteristic polynomial $\mathcal{P}^f(v) = 0$ has real roots satisfying $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$ and $|v_4| > 1$. The fifth and the sixth roots of $\mathcal{P}^b(v) = 0$ are $v_5 = \sqrt{\rho_\epsilon}$ and $v_6 = -\sqrt{\rho_\epsilon}$. Clearly if $|\rho_\epsilon| > 1$ then $|v_5| > 1$ and $|v_6| > 1$ whereas if $|\rho_\epsilon| < 1$ then $|v_5| < 1$ and $|v_6| < 1$. Therefore using this, the characterization of the roots of $\mathcal{P}^f(v) = 0$ and (58) we can conclude the following. If $|\rho_\epsilon| > 1$ then $\mathcal{P}^b(v) = 0$ has four explosive roots namely $|v_2| > 1$, $|v_4| > 1$, $|v_5| > 1$ and $|v_6| > 1$. While if $|\rho_\epsilon| < 1$ then $\mathcal{P}^b(v) = 0$ has two explosive roots namely $|v_2| > 1$ and $|v_4| > 1$. Therefore if $|\rho_\epsilon| > 1$ the number of explosive roots is equal to the number of non-predetermined variables ($\hat{\epsilon}_t$, $\hat{c}_t^T$, $\hat{\pi}_t^N$ and $\hat{c}_t^N$). Hence applying the results of Blanchard and Kahn (1980) it follows that there exists a unique equilibrium. This completes the proof for a).

On the contrary by the previous analysis if $|\rho_\epsilon| < 1$ the number of explosive roots, 2, is less than the number of non-predetermined variables ($\hat{\epsilon}_t$, $\hat{c}_t^T$, $\hat{\pi}_t^N$ and $\hat{c}_t^N$), 4. Applying the results of Blanchard and Kahn (1980) it follows that there exists an infinite number of perfect foresight equilibria converging to the steady state. The degree of indeterminacy is the difference between the number of non-predetermined variables and the number of explosive roots. This completes the proof for b). ■

A.1.4 Lemmata and Proofs for the Results in the Simple Model

Lemma 1 *Consider the characteristic polynomial* $\mathcal{P}\ (v)=v^2+Tv+D=0$. *If either a)* $P(1)<0$ *or b)* $P(-1)<0$ *then the roots are real.*

Proof. First recall from Azariadis (1993) that a sufficient condition to have real roots is that $T^2-4D\geq 0$. To prove a) note that $\mathcal{P}(1)<0$ means that $\mathcal{P}(1)=1-T+D<0$. But this implies that $4T-4>4D$ which in turn leads to $T^2-4D>T^2-4T+4=(T-2)^2\geq 0$. Hence the roots are real. Next we prove b). $\mathcal{P}(-1)<0$ means that $\mathcal{P}(1)=1+T+D<0$. But this implies that $-4T-4>4D$ that in turn leads to $T^2-4D>T^2+4T+4=(T+2)^2\geq 0$. Hence the roots are real. ■

Lemma 2 *The roots of the characteristic polynomial*

$$\mathcal{P}_{bc^T}(v)=v^2-\left(1+\frac{1+\psi}{\beta}+\psi\kappa\xi^T\right)v+\frac{1+\psi}{\beta}=0$$

are real and satisfy $|v_1|<1$ *and* $|v_2|>1$.

Proof. First using the definition of $\mathcal{P}_{bc^T}(v)$, Assumptions 1, 2 and 3 and definitions in (22) we obtain that

$$\mathcal{P}_{bc^T}(1)=-\psi\kappa\xi^T<0$$
$$\mathcal{P}_{bc^T}(-1)=2\left(1+\frac{1+\psi}{\beta}\right)+\psi\kappa\xi^T>0$$

Since $\mathcal{P}_{bc^T}(1)<0$ then by Lemma 1 we know that the two roots are real. In addition from Azariadis (1993), having $\mathcal{P}_{bc^T}(1)<0$ and $\mathcal{P}_{bc^T}(-1)>0$ imply that one root lies inside of the unit circle and the other one lies outside the unit circle. Without loss of generality we can conclude that $|v_1|<1$ and $|v_2|>1$. ■

Lemma 3 *The roots of the characteristic polynomial*

$$\mathcal{P}_{\pi^N c^N}(v)=v^2-\left(1+\frac{1}{\beta}+\varphi\xi^N\right)v+\frac{1}{\beta}=0$$

are real and satisfy $|v_3|<1$ *and* $|v_4|>1$.

Proof. First using the definition of $\mathcal{P}_{\pi^N c^N}(v)$, Assumptions 1, 2 and 3 and definitions in (22) we obtain that: $\mathcal{P}_{\pi^N c^N}(-1)=-\varphi\xi^N<0$ and $\mathcal{P}_{\pi^N c^N}(1)=2\left(1+\frac{1}{\beta}\right)+\varphi\xi^N>0$. Since $\mathcal{P}_{\pi^N c^N}(-1)<0$ then by Lemma 1 we know that the two roots of $\mathcal{P}_{\pi^N c^N}(v)=0$ are real. In addition from Azariadis (1993), having $\mathcal{P}_{\pi^N c^N}(-1)<0$ and $\mathcal{P}_{\pi^N c^N}(1)>0$ imply that one root lies inside of the unit circle and the other one lies outside the unit circle. Without loss of generality we can conclude that that $|v_3|<1$ and $|v_4|>1$. ■

Lemma 4 *The roots of the characteristic polynomial*

$$\mathcal{P}^f(v)=\mathcal{P}_{bc^T}(v)\mathcal{P}_{\pi^N c^N}(v)=0 \tag{59}$$

where

$$\mathcal{P}_{bc^T}(v) = v^2 - \left(1 + \frac{1+\psi}{\beta} + \psi\kappa\xi^T\right)v + \frac{1+\psi}{\beta}$$
$$\mathcal{P}_{\pi^N c^N}(v) = v^2 - \left(1 + \frac{1}{\beta} + \varphi\xi^N\right)v + \frac{1}{\beta}$$

are real and satisfy $|v_1| < 1$, $|v_2| > 1$, $|v_3| < 1$ *and* $|v_4| > 1$.

Proof. By Lemma 2 we know that the characteristic polynomial $\mathcal{P}_{bc^T}(v) = 0$ has two real roots satisfying $|v_1| < 1$ and $|v_2| > 1$. On the other hand by Lemma 3 we know that the characteristic polynomial $\mathcal{P}_{\pi^N c^N}(v) = 0$ has two real roots satisfying $|v_3| < 1$ and $|v_4| > 1$. Therefore from these Lemmata and the definition of $\mathcal{P}^f(v) = 0$ the result of the Lemma follows. ■

A.2 The Augmented Model

A.2.1 Forward-Looking and Backward-Looking Policies in the Augmented Model

We pursue the determinacy of equilibrium analysis for forward-looking policies ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$) and backward looking policies ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$). We use the parametrization of Table 2, and as an illustrative case we focus on the experiment of characterizing the equilibrium while we vary the degree of responsiveness to future (past) currency depreciation (ρ_ϵ) and the intratemporal elasticity of substitution (a) keeping the rest constant. The results are presented in Figure 5. The top panel shows the results for a forward-looking policy whereas the bottom panel presents the results for backward-looking policies.

From the top-panel we can infer that forward-looking policies always induce multiple cyclical equilibria as long as $\rho_\epsilon \neq 0$ and either $|\rho_\epsilon| > 1$ or $|\rho_\epsilon| < 1$. On the other hand, for backward-looking rules, the coefficient of response to past depreciation, ρ_ϵ, plays an important role in the characterization of the equilibrium. In particular timid rules with respect to past depreciation ($|\rho_\epsilon| < 1$) always induce multiple equilibria regardless of the intratemporal elasticity of substitution (a). While aggressive rules ($|\rho_\epsilon| > 1$) can guarantee a unique equilibrium. Nevertheless being aggressive with respect to past depreciation ($|\rho_\epsilon| > 1$) is not a sufficient condition to guarantee a unique equilibrium. It is only a necessary condition.

As before varying other structural parameters different from the intratemporal elasticity of substitution (a) in tandem with ρ_ϵ lead to similar results.[42] The following proposition summarizes these results.

Proposition 5 *Under a crisis when the collateral constraint is binding,*

1. *If the government follows a forward-looking policy* ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t+1}$ *with* $\rho_\epsilon \neq 0$ *and either* $|\rho_\epsilon| > 1$ *or* $|\rho_\epsilon| < 1$), *then there exists a continuum of "cyclical" perfect foresight equilibria in which the sequences* $\{\hat{c}_t^T,\ \hat{c}_t^N,\ \hat{\zeta}_t,\ \hat{h}_t^T,\ \hat{h}_t^N,\ \hat{I}_t,\ \hat{b}_t^*,\ \hat{mc}_t,\ \hat{e}_t,\ \hat{q}_t^T,\ \hat{q}_t^N,\ \hat{w}_t,\ \hat{\epsilon}_t,\ \hat{\pi}_t^N,\ \hat{R}_t\}_{t=0}^\infty$ *converge asymptotically to the steady state.*

[42] The results are available from the author upon request.

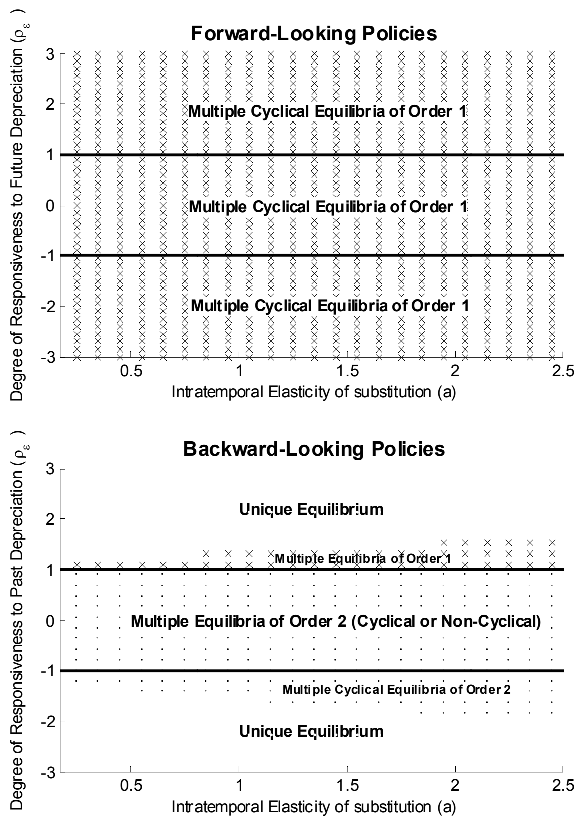

Figure 5: This Figure shows the characterization of the equilibrium for forward-looking (top-panel) and backward-looking (bottom-panel) policies varying the degree of responsiveness to currency depreciation (ρ_ϵ) and the intratemporal elasticity of substitution (a). It is assumed that $\rho_\epsilon \neq -1, 0, 1$. A cross "x" denotes parameter combinations under which the policy induces multiple equilibria whose degree of indeterminacy is of order one. A dot "." represents parameter combinations under which the policy induces multiple equilibria whose degree of indeterminacy is of order two. In the Figure it is also specified whether these multiple equilibria are cyclical or non-cyclical. The white regions represent parameter combinations under which there exists a unique equilibrium.

2. If the government follows a backward-looking policy ($\hat{R}_t = \rho_\epsilon \hat{\epsilon}_{t-1}$ with $\rho_\epsilon \lesseqgtr 0$),

a) *if $|\rho_\epsilon| < 1$ then there exists a continuum of perfect foresight equilibria, possibly "cyclical", in which the sequences $\{\hat{c}_t^T, \hat{c}_t^N, \hat{\zeta}_t, \hat{h}_t^T, \hat{h}_t^N, \hat{I}_t, \hat{b}_t^*, \hat{mc}_t, \hat{e}_t, \hat{q}_t^T, \hat{q}_t^N, \hat{w}_t, \hat{\epsilon}_t, \hat{\pi}_t^N, \hat{R}_t\}_{t=0}^\infty$ converge asymptotically to the steady state.*

b) *$|\rho_\epsilon| > 1$ is a necessary but not sufficient condition for the existence of a unique perfect foresight equilibrium where the sequences $\{\hat{c}_t^T, \hat{c}_t^N, \hat{\zeta}_t, \hat{h}_t^T, \hat{h}_t^N, \hat{I}_t, \hat{b}_t^*, \hat{mc}_t, \hat{e}_t, \hat{q}_t^T, \hat{q}_t^N, \hat{w}_t, \hat{\epsilon}_t, \hat{\pi}_t^N, \hat{R}_t\}_{t=0}^\infty$ converge asymptotically to the steady state.*

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